NATIONAL INSTITUTE OF TECHNOLOGY SRINAGAR

Subject: Heat Transfer and Fluid Flow **Dr. H. S. Pali**

NAME OF DEPARTMENT : Metallurgical & Materials Engineering

1. Subject Code: MEC-412 **Course Title:** Heat Transfer and Fluid Flow

Objective:

To understand the basic principles of heat transfer and fluid flow.

UNIT 1

Modes and Laws of heat transfer. **CONDUCTION:** Steady State and unsteady state. Heat flow through composite walls. Heating and cooling of plates, cylinders and spheres. **CONVECTION:** Free and forced convection. Reynolds, Grasshoofs, Nusselt and Station numbers.

UNIT 2

RADIATION: Emissivity, absorptivity, reflectivity and transmissivity. Simple Heat transfer etween black and gray surfaces. Re-radiating surfaces. Heat losses from furnaces. Combined effects of conduction, convection and radiation. Steady and unsteady heat flow in some metallurgical processes, e.g melting, solidification, heating/cooling of ingots and billets, etc.

UNIT 3

FLUID FLOW: Viscosity and Newton’s law of viscosity. Newtonian and non-Newtonian fluids. Conservation of mass and continuity equation. Energy of fluids. Euler’s and Bernoullis equations. Loss of energy due to friction. Flow through pipes. Laminar and Turbulent Flow, Reynold’s number. Compressed air and air blasts. Energy used for compression. Compressor and blower efficiency characteristics.

Introduction

1.1 Difference between heat and temperature

In heat transfer problems, we often interchangeably use the terms heat and temperature Actually, there is a distinct difference between the two. **Temperature** is a measure of the amount of energy possessed by the molecules of a substance. It manifests itself as a degree of hotness, and can be used to predict the direction of heat transfer. The usual symbol for temperature is T. The scales for measuring temperature in SI units are the ℃ celsius and Kelvin temperature scales.

Heat, on the other hand, is energy in transit. Spontaneously, heat flows from a hotter body to a colder one. The usual symbol for heat is Q. In the SI system, commonunits for measuring heat are the Joule and calorie.

1.2 Difference between thermodynamics and heat transfer

Thermodynamics tells us:

- how much heat is transferred (ΔQ)
- how much work is done (ΔW)
- final state of the system

Heat transfer tells us:

- How (with what **modes**) ΔQ is transferred
- At what **rate** ΔQ is transferred
- Temperature distribution inside the body

1. Introduction

Heat transfer is a science, which deals with the flow of heat from a higher temperature to lower temperature. Heat cannot be stored and it is defined as the energy in transit due to the difference in the temperatures of the hot and cold bodies. The study of heat transfer not only explains how the heat energy transports but also predicts about the rate of heat transfer. When a certain amount of water is evaporated or condensed, the amount of heat transferred in either of the processes is same. However, the rate of heat transfer in both the cases may be different.

At this point, it is very important to understand about the basic information that the phases of a substance (solid, liquid, and gas) are associated with its energy content. In the solid phase, the molecules or atoms are very closely packed to give a rigid structure (fig.1.1a). In the liquid phase, sufficient thermal energy is present, which keeps the molecules sufficiently apart and as a result the rigidity looses (fig.1.1b). In the gas phase, the presence of additional energy results in a complete separation and the molecules or atoms are free to move anywhere in the space (fig.1.1c). It must be noticed that whenever a change in phase occurs, a large amount of energy involves in the transition.

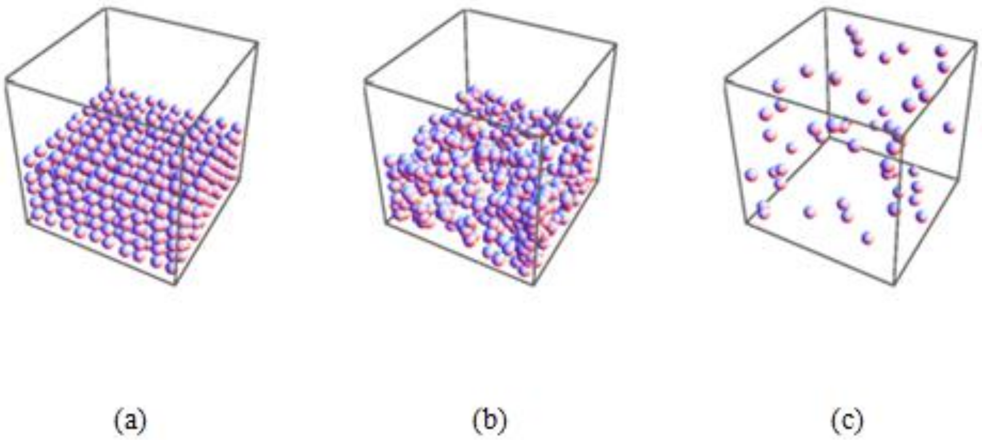


Fig. 1.1: Relative molecular distance of different phases of a substance at a fixed temperature

(a) gas/vapour, (b) liquid, and (c) solid

As we are dealing with the heating and cooling of materials in almost our all the processes, the heat transfer is an indispensable part of any of the industries. Therefore, heat transfer is a common subject in many engineering disciplines, especially mechanical and chemical engineering. Study of heat transfer has a vital role in the chemical process industries. Chemical engineers must have a thorough knowledge of heat transfer principles and their applications.

Heat flows from higher temperature to lower temperature. Though it looks simple, heat transfer is a quite complex phenomenon.

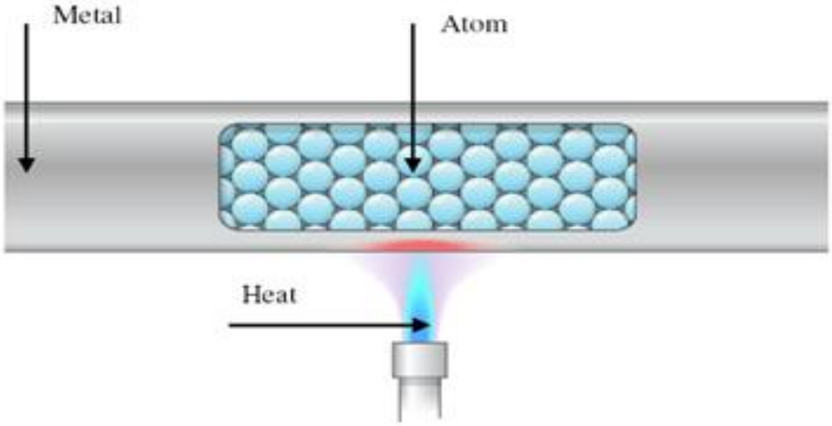
There are three different modes in which heat may pass from a hot body to a cold one. These modes are conduction, convention, and radiation. It should be noted that the heat transfer takes place in combination of two or three modes in any of the real engineering application. In this chapter, we will briefly discuss about the different modes of heat transfer along with the various basic information that will help us as a building block for further study.

1.1 Mode of heat transfer

In this section, we will discuss about the three different modes of heat transfer. The discussion will help us to understand about the conduction, convection, and radiation. Moreover, we would be able to understand the basic difference between the three modes of heat transfer.

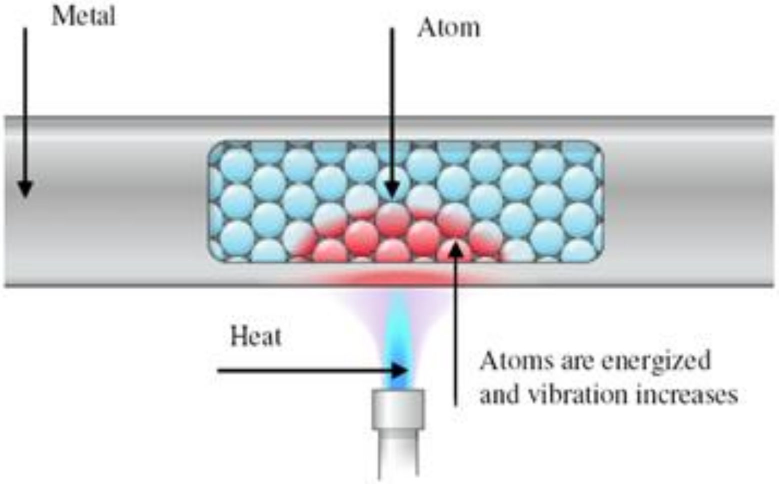
1.1.1 Conduction

Conduction is the transfer of heat in a continuous substance without any observable motion of the matter. Thus, heat conduction is essentially the transmission of energy by molecular motion. Consider a metallic rod being heated at the end and the other end of the rod gets heated automatically. The heat is transported from one end to the other end by the conduction phenomenon. The molecules of the metallic rod get energy from the heating medium and collide with the neighbouring molecules. This process transfers the energy from the more energetic molecules to the low energetic molecules. Thus, heat transfer requires a temperature gradient, and the heat energy transfer by conduction occurs in the direction of decreasing temperature. Figure 1.2 shows an illustration for the conduction, where the densely packed atoms of the rod get energized on heating and vibration effect transfers the heat as described in fig.1.2.



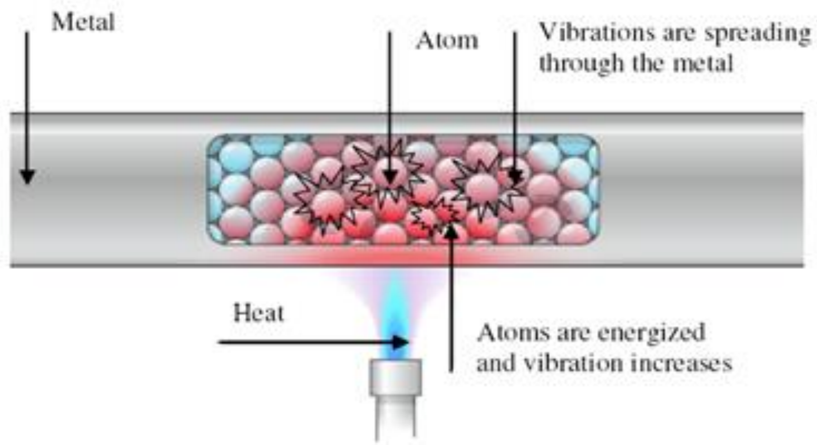
(a)

Step 1: The metallic rod is being heated



(b)

Step 2: The energy is transferred to the atoms and vibration of the atoms started



(c)

Step 3: As more energy is transferred to the atoms, the vibration increases as well as transferred to the other atoms

Fig.1.2: Different stages during conduction in a metallic rod

Fig.1.2: Different stages during conduction in a metallic rod

1.1 Mode of heat transfer

In this section, we will discuss about the three different modes of heat transfer. The discussion will help us to understand about the conduction, convection, and radiation. Moreover, we would be able to understand the basic difference between the three modes of heat transfer.

1.1.1 Conduction

Conduction is the transfer of heat in a continuous substance without any observable motion of the matter. Thus, heat conduction is essentially the transmission of energy by molecular motion. Consider a metallic rod being heated at the end and the other end of the rod gets heated automatically. The heat is transported from one end to the other end by the conduction phenomenon. The molecules of the metallic rod get energy from the heating medium and collide with the neighboring molecules. This process transfers the energy from the more energetic molecules to the low energetic molecules. Thus, heat transfer requires a temperature gradient, and the heat energy transfer by conduction occurs in the direction of decreasing temperature. Figure 1.2 shows an illustration for the conduction, where the densely packed atoms of the rod get energized on heating and vibration effect transfers the heat as described in fig.1.2.

Conduction takes place at a microscopic level. Atoms or molecules at higher temperature have high levels of energy. Through vibration, this energy is passed on to neighboring atoms and molecules. In other words, in conductive mode of heat transfer, vibrating atoms and molecules a part of their energy. This kind of heat transfer can take place between two or more substances or through the substance. Conduction can also take place when electrons move from one atom to another. Transient conduction takes place when temperature within an object changes a the function of time.

1.1.2 Convection

When a macroscopic particle of a fluid moves from the region of hot to cold region, it carries with it a definite amount of enthalpy. Such a flow of enthalpy is known as convection. Convection may be natural or forced. In natural convection, the movement of the fluid particles is due to the buoyancy forces generated due to density difference of heated and colder region of the fluid as shown in the fig.1.3a. Whereas, in forced convection the movement of fluid particles from the heated region to colder region is assisted by some mechanical means too (eg., stirrer) as shown in fig.1.3b.

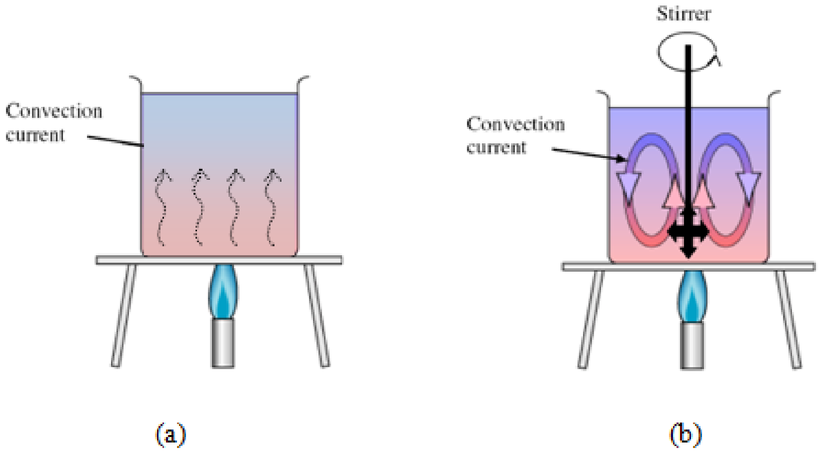


Fig.1.3: Heat transfer through convection (a) natural, and (b) forced

Convection is a mode of heat transfer which takes place through the movement of collective masses of heated atoms and molecules. Convection requires actual flow of material particles whereas in conduction, the heat is transferred through vibration without the atoms or molecules leaving their original position. In convection, heat transfer takes place through both diffusion and advection. As convection requires the actual movement of the heated atoms/ molecules, it requires presence of a fluid for heat transfer.

1.1.3 Radiation

We have seen that a medium is required for the heat transfer in case of conduction and convection. However, in case of radiation, electromagnetic waves pass through the empty space. Electromagnetic waves travel at the velocity of light in vacuum. These waves are absorbed, reflected, and/or transmitted by the matter, which comes in the path of the wave. We will limit our discussion (in this NPTEL course) to the thermal radiation. Thermal radiation is the term used to describe the electromagnetic radiation, which is observed to be emitted by the surface of the thermally excited body. The heat of the Sun is the most obvious example of thermal radiation.

There will be a continuous interchange of energy between two radiating bodies, with a net exchange of energy from the hotter to the colder body as shown in the fig.1.4.

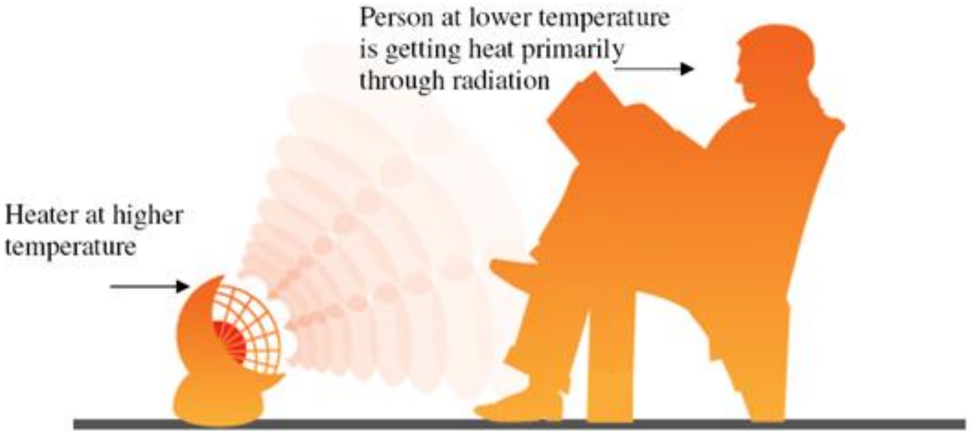
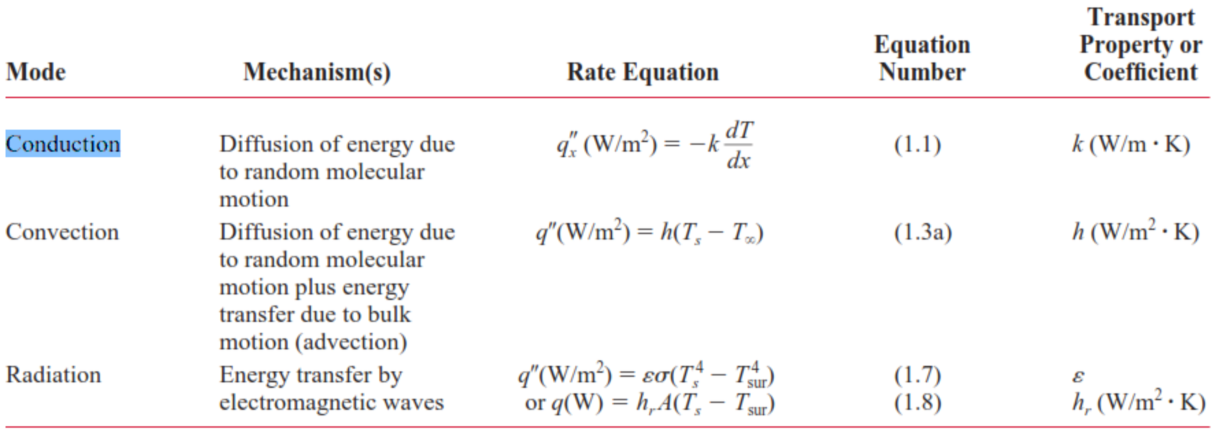


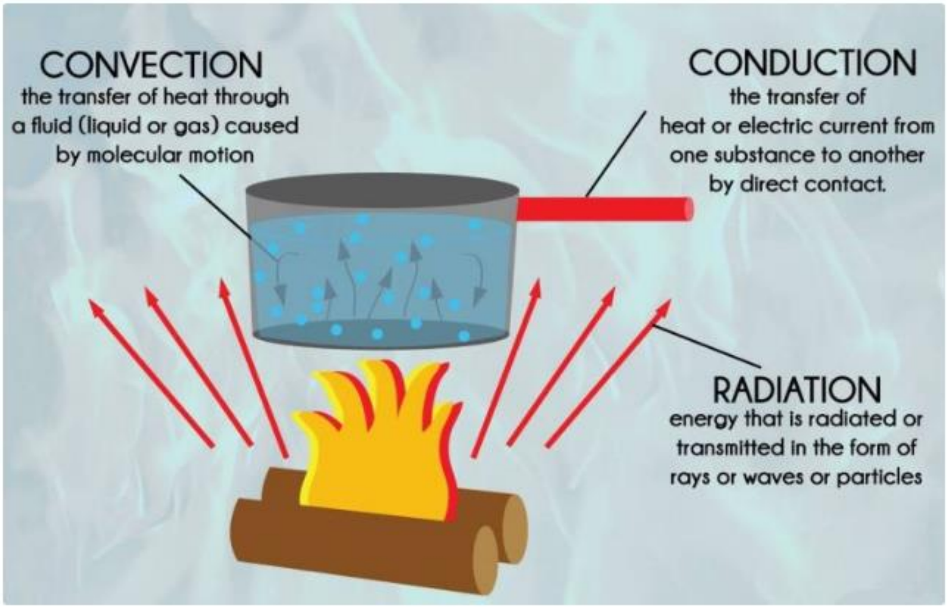
Fig.1.4: Heat transfer through radiation

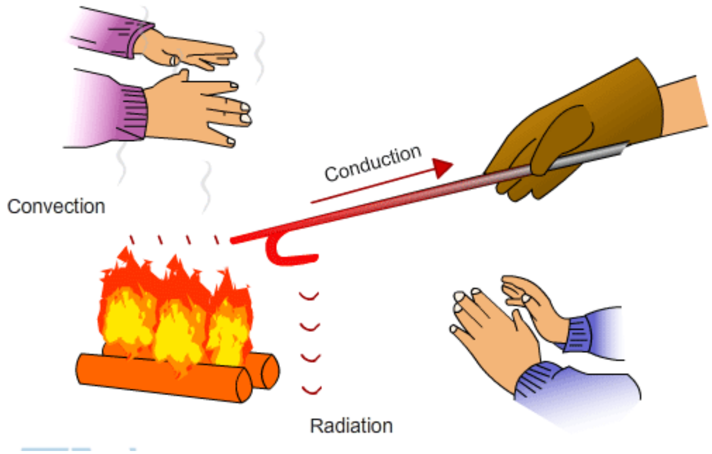
Radiation is a mode of heat transfer which takes place through vacuum and hence, does not need a physical medium. Radiation takes place either through vacuum or through a transparent medium. In radiative mode, heat transfer takes place through photons present in the electromagnetic waves. The random movement of atoms and molecules in heated substances results in emission of electromagnetic waves which carry the heat to be transferred. The radiative heat transfer is governed by Stephen-Boltzman law. A body radiates heat at all temperatures above the absolute zero, irrespective of the ambient temperature.



Mode	Mechanism(s)	Rate Equation	Equation Number	Transport Property or Coefficient
Conduction	Diffusion of energy due to random molecular motion	$q''_x\ (\mathrm{W/m^2}) = -k\dfrac{dT}{dx}$	(1.1)	k (W/m · K)
Convection	Diffusion of energy due to random molecular motion plus energy transfer due to bulk motion (advection)	$q''(\mathrm{W/m^2}) = h(T_s - T_\infty)$	(1.3a)	h (W/m^2 · K)
Radiation	Energy transfer by electromagnetic waves	$q''(\mathrm{W/m^2}) = \varepsilon\sigma(T_s^4 - T_{\mathrm{sur}}^4)$ or $q(\mathrm{W}) = h_r A(T_s - T_{\mathrm{sur}})$	(1.7) (1.8)	ε h_r (W/m^2 · K)

Modes and Laws of Heat Transfer




Convection
Conduction
Radiation

NATIONAL INSTITUTE OF TECHNOLOGY SRINAGAR

Subject: Heat Transfer and Fluid Flow **Dr. H. S. Pali**

NAME OF DEPARTMENT : Metallurgical & Materials Engineering

UNIT 1

Modes and Laws of heat transfer. **CONDUCTION:** Steady State and unsteady state. Heat flow through composite walls. Heating and cooling of plates, cylinders and spheres. **CONVECTION:** Free and forced convection. Reynolds, Grasshoofs, Nusselt and Station numbers.

Conduction

Conduction may be viewed as the transfer of energy from the more energetic to the less energetic particles of a substance due to interactions between the particles. It is concept of *atomic* and *molecular activity.*

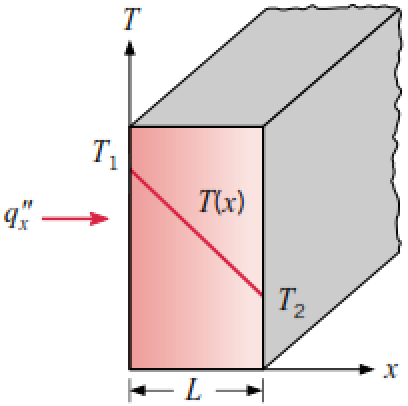
Higher temperatures are associated with higher molecular energies, and when neighboring molecules collide, as they are constantly doing, a transfer of energy from the more energetic to the less energetic molecules must occur. In the presence of a temperature gradient, energy transfer by conduction must then occur in the direction of decreasing temperature. Collisions between molecules enhance this energy transfer. We may speak of the net transfer of energy by random molecular motion as a diffusion of energy.

The situation is much the same in liquids (or any fluid), although the molecules are more closely spaced and the molecular interactions are stronger and more frequent. Similarly, in a solid, conduction may be attributed to atomic activity in the form of **lattice vibrations**. The modern view is to ascribe the energy transfer to lattice waves induced by atomic motion. In an electrical nonconductor, the energy transfer is exclusively via these lattice waves; in a conductor it is also due to the translational motion of the free electrons.

Examples of conduction heat transfer are legion (crowed or mass). The exposed end of a metal spoon suddenly immersed in a cup of hot coffee will eventually be warmed due to the conduction of energy through the spoon. On a winter day there is significant energy loss from a heated room to the outside air. This loss is principally due to conduction heat transfer through the wall that separates the room air from the outside air.

It is possible to quantify heat transfer processes in terms of appropriate rate equations. These equations may be used to compute the amount of energy being transferred per unit time. For heat conduction, the rate equation is known as Fourier's law. For the one-dimensional plane wall shown in Figure 1, having a temperature distribution T(x), the rate equation is expressed as

$$q'' = -k\frac{dT}{dx} \quad \text{..............................1}$$



The heat flux q" (W/m^2) is the heat transfer rate in the x direction per unit area perpendicular to the direction of transfer, and it is proportional to the temperature gradient, dT/dx, in this direction. The parameter k is a transport property known as the thermal conductivity (W/m.K) and is a characteristic of the wall material. The minus sign is a consequence of the fact that heat is transferred in the direction of decreasing temperature (Second law). Under the steady-state conditions shown in Figure 1, where the temperature distribution is linear, the temperature gradient may be expressed as

$$\frac{dT}{dx} = \frac{T_2 - T_1}{L}$$

And the heat flux is then

$$q'' = -k\frac{T_2 - T_1}{L}$$
$$q'' = -k\frac{T_2 - T_1}{L} = k\frac{\Delta T}{L}$$

Note that this equation provides a heat flux, that is, the rate of heat transfer per unit area. The heat rate by conduction, q_x (W), through a plane wall of area A is then the product of the flux and the area, $q_x = q''A$

1.2 Material properties of importance in heat transfer

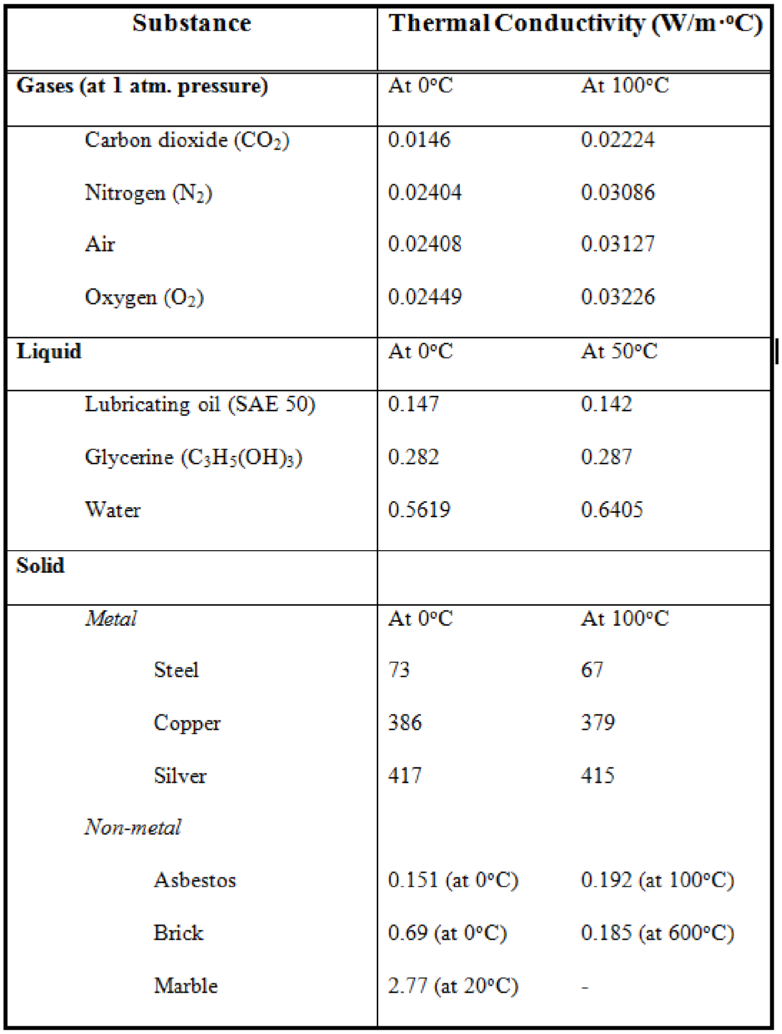
Before understanding heat transfer laws, we have to understand various properties of the material. This section is devoted to a brief discussion of some of the important properties of the material.

1.2.1 Thermal conductivity

As discussed earlier, the heat conduction is the transmission of energy by molecular action. Thermal conductivity is the property of a particular substance and shows the ease by which the process takes place. Higher the thermal conductivity more easily will be the heat conduction through the substance. It can be realized that the thermal conductivity of a substance would be dependent on the chemical composition, phase (gas, liquid, or solid), crystalline structure (if solid), temperature, pressure, and its homogeneity.

The thermal conductivity of various substances is shown in table-1.

Table-1.1: Thermal conductivities of various substances at 0oC



Substance	Thermal Conductivity (W/m·°C)	
Gases (at 1 atm. pressure)	At 0°C	At 100°C
Carbon dioxide (CO_2)	0.0146	0.02224
Nitrogen (N_2)	0.02404	0.03086
Air	0.02408	0.03127
Oxygen (O_2)	0.02449	0.03226
Liquid	At 0°C	At 50°C
Lubricating oil (SAE 50)	0.147	0.142
Glycerine ($C_3H_5(OH)_3$)	0.282	0.287
Water	0.5619	0.6405
Solid		
Metal	At 0°C	At 100°C
Steel	73	67
Copper	386	379
Silver	417	415
Non-metal		
Asbestos	0.151 (at 0°C)	0.192 (at 100°C)
Brick	0.69 (at 0°C)	0.185 (at 600°C)
Marble	2.77 (at 20°C)	-

The general results of the careful analysis of the table-1.1 is as follows,

- Thermal conductivity depends on the chemical composition of the substance.
- Thermal conductivity of the liquids is more than the gasses and the metals have the highest.
- Thermal conductivity of the gases and liquids increases with the increase in temperature.
- Thermal conductivity of the metal decreases with the increase in temperature.
- Thermal conductivity is affected by the phase change.

These differences can be explained partially by the fact that while in gaseous state, the molecules of a substance are spaced relatively far away and their motion is random. This means that energy transfer by molecular impact is much slower than in the case of a liquid, in which the motion is still random but in liquids the molecules are more closely packed. The same is true concerning the difference between the thermal conductivity of the liquid and solid phases. However, other factors are also important when the solid state is formed.

Solid having a crystalline structure has high thermal conductivity than a substance in an amorphous solid state. Metal, crystalline in structure, have greater thermal conductivity than non-metal (refer table-1.1). The irregular arrangement of the molecules in amorphous solids inhibits the effectiveness of the transfer of the energy by molecular impact. Therefore, the thermal conductivity of the non-metals is of the order of liquids. Moreover, in solids, there is an additional transfer of heat energy resulting from vibratory motion of the crystal lattice as a whole, in the direction of decreasing temperature.

Many factors are known to influence the thermal conductivity of metals, such as chemical composition, atomic structure, phase changes, grain size, temperature, and pressure. Out of the above factors, the temperature, pressure, and chemical composition are the most important. However, if we are interested in a particular material then only the temperature effects has to be accounted for.

As per the previous discussion and the table it is now clear that the thermal conductivity of the metal is directly proportional to the absolute temperature and mean free path of the molecules. The mean free path decreases with the increase in temperature so that the thermal conductivity decreases with the temperature. It should be noted that it is true for the pure metal, and the presence of impurity in the metal may reverse the trend. It is usually possible to represent the thermal conductivity of a metal by a linear relation $k = k_o(1 + bT)$, where k_o is the thermal conductivity of the metal at 0^oC, T is the absolute temperature, and b is a constant.

In general the thermal conductivity of the liquids is insensitive to the pressure if the pressure is not very close to the critical temperature. Therefore, in liquids (as in solids) the temperature effects on the thermal conductivity are generally considered. Liquids, in general, exhibit a decreasing thermal conductivity with temperature. However, water is a notable exception. Water has the highest thermal conductivity among the non-metallic liquids, with a maximum value occurring at 450^oC.

The thermal conductivity of a gas is relatively independent of pressure if the pressure is near 1 atm. Vapours near the saturation point show strong pressure dependence. Steam and air are of great engineering importance. Steam shows irregular behaving rather showing a rather strong pressure dependence for the thermal conductivity as well as temperature dependence.

The above discussions concerning thermal conductivity were restricted to materials composed of homogeneous or pure substances. Many of the engineering materials encountered in practice are not of this nature like building material, and insulating material. Some material may exhibit non-isotropic conductivities. The non-isotropic material shows different conductivity in different direction in the material. This directional preference is primarily the result of the fibrous nature of the material like wood, asbestos etc.

1.2.2 Specific heat capacity

Now we know that the thermal conductivity facilitates the heat to propagate through the material due to the temperature gradient. Similarly, specific heat capacity or specific heat is the capacity of heat stored by a material due to variation in temperature. Thus the specific heat capacity (unit: kJ/kg·^{o}C) is defined as the amount of thermal energy required to raise the temperature of a unit amount of material by 1^{o}C. Since heat is path dependent, so is specific heat. In general, the heat transfer processes used in the chemical process plant are at constant pressure; hence the specific heat capacity (c_o) is generally used.

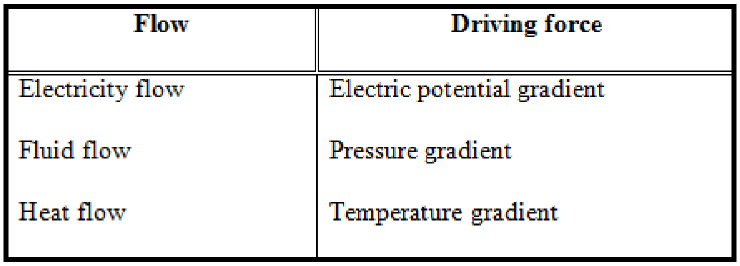
Frequently Asked Questions (Module 1)

Q.1. What is the basic difference among conduction, convection, and radiation?
Q.2. Define thermal conductivity.
Q.3. What is the order of thermal conductivity of gas, liquid, and metal in general?
Q.4. What should be the approach to select a good thermal insulator?
Q.5. Discuss the effect of temperature on thermal conductivity.
Q.6. What is the difference between thermal conductivity and specific heat capacity?

Conduction: One Dimensional

The fundamentals of heat conduction were established over one and a half century and its contribution goes to a French mathematician and physicist, Jean Baptiste Joseph Fourier. You may be aware that any flow whether it is electricity flow, fluid flow, or heat flow needs a driving force. The flow is proportional to the driving force and for various kinds of flows the driving force is shown in the table 2.1.

Table 2.1. Various flows and their driving forces



Flow	Driving force
Electricity flow	Electric potential gradient
Fluid flow	Pressure gradient
Heat flow	Temperature gradient

Thus the heat flow per unit area per unit time (heat flux, q') can be represented by the following relation,

$$\dot{q}' \propto \frac{dT}{dx} \qquad (2.1)$$

$$\dot{q}' = k\frac{dT}{dx} \qquad (2.2)$$

where, proportionality constant *k* is the thermal conductivity of the material, *T* is the temperature and *x* is the distance in the direction of heat flow. This is known as Fourier's law of conduction.

The term steady-state conduction is defined as the condition which prevails in a heat conducting body when temperatures at fixed points do not change with time. The term one-dimensional is applied to a heat conduction problem when only one coordinate is required to describe the distribution of temperature within the body. Such a situation hardly exists in real engineering problems. However, by considering one-dimensional assumption the real problem is solved fairly upto the accuracy of practical engineering interest.

2.1 Steady-state conduction through constant area

A simple case of steady-state, one-dimensional heat conduction can be considered through a flat wall as shown in the fig.2.1.

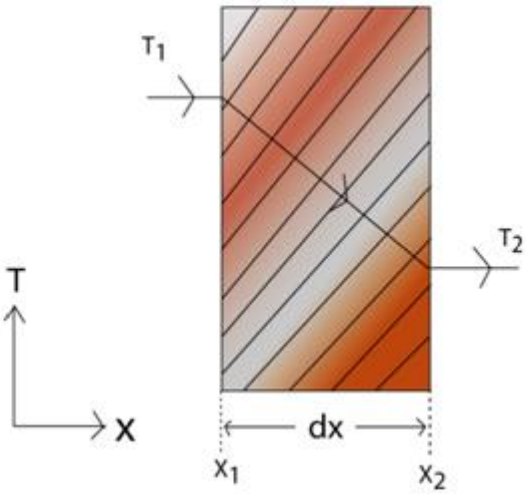


Fig.2.1: Steady-state conduction through a slab (constant area)

The flat wall of thickness d_x is separated by two regions, the one region is at high temperature (T_1) and the other one is at temperature T_2. The wall is very large in comparison of the thickness so that the heat losses from the edges are negligible. Consider there is no generation or accumulation of the heat in the wall and the external surfaces of the wall are at isothermal temperatures T_1 and T_2. The area of the surface through which the heat transfer takes place is A. Then the eq.2.2 can be written as,

$$\dot{q}' = -k\frac{dT}{dx} \tag{2.3}$$

$$\frac{\dot{q}}{A} = -k\frac{dT}{dx} \tag{2.4}$$

$$\frac{\dot{q}}{A} = -k\frac{T_2 - T_1}{x_2 - x_1} \tag{2.5}$$

The negative sign shows that the heat flux is from the higher temperature surface to the lower temperature surface and is the rate of heat transfer through the wall.

Now if we consider a plane wall made up of three different layers of materials having different thermal conductivities and thicknesses of the layers, the analysis of the conduction can be done as follows.

Consider the area (A) of the heat conduction (fig.2.2) is constant and at steady state the rate of heat transfer from layer-1 will be equal to the rate of heat transfer from layer-2. Similarly, the rate of heat transfer through layer-2 will be equal to the rate of heat transfer through layer-3. If we know the surface temperatures of the wall are maintained at T_1 and T_2 as shown in the fig.2.2, the temperature of the interface of layer1 and layer 2 is assumed to be at T' and the interface of layer-2 and layer-3 as T''.

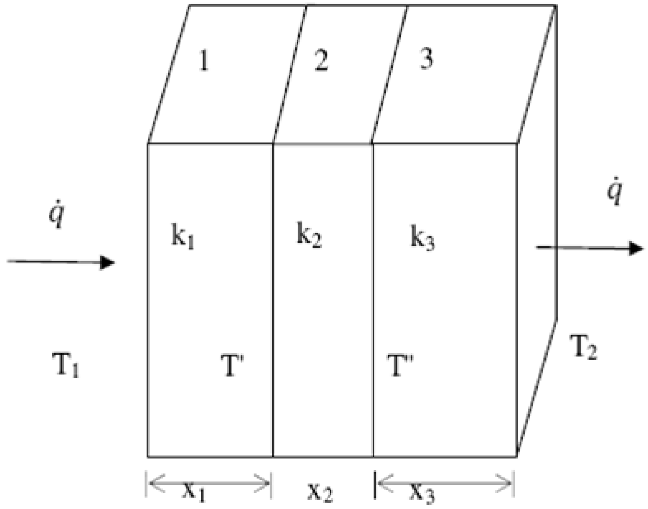


Fig.2.2: Heat conduction through three different layers

The rate of heat transfer through layer-1 to layer-2 will be,

$$\dot{q} = \frac{k_1 A\,(T_1 - T')}{x_1} \quad or \quad (T_1 - T') = \frac{\dot{q}}{1/(x_1/k_1 A)} \tag{2.6}$$

and,

The rate of heat transfer through layer 2 to layer 3 will be,

$$\dot{q} = \frac{k_2 A\,(T' - T'')}{x_2} \quad or \quad (T' - T'') = \frac{\dot{q}}{1/(x_2/k_2 A)} \tag{2.7}$$

and,

The rate of heat transfer through layer 3 to the other side of the wall,

$$\dot{q} = \frac{k_3 A\,(T'' - T_2)}{x_3} \quad or \quad (T'' - T_2) = \frac{\dot{q}}{1/(x_3/k_3 A)} \tag{2.8}$$

On adding the above three equations,

$$\dot{q} = \frac{T_1 - T_2}{\frac{x_1}{k_1 A} + \frac{x_2}{k_2 A} + \frac{x_3}{k_3 A}} \tag{2.9}$$

$$\dot{q} = \frac{T_1 - T_2}{R_1 + R_2 + R_3} \tag{2.10}$$

Where, R represents the thermal resistance of the layers. The above relation can be written analogous to the electrical circuit as,

$$Rate\ of\ heat\ flow = \frac{Temperaure\ difference}{Resistance} \qquad (2.11)$$

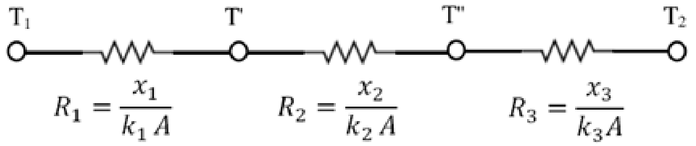


Fig 2.3: Equivalent electrical circuit of the fig.2.2

The wall is composed of 3-different layers in series and thus the total thermal resistance was represented by R (= $R_1 + R_2 + R_3$). The discussed concept can be understood by the illustrations shown below.

The unit of the various parameters used above is summarized as follows,

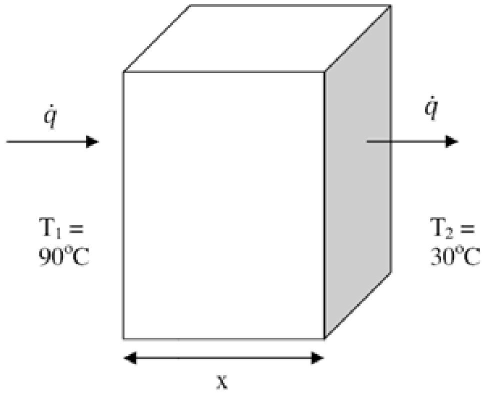
Parameter	Symbol used	Unit
Heat flux	$\dot{q}'$	W/m^2 or J/(s·m^2)
Heat flow rate	$\dot{q}$	W or J/s
Thermal conductivity	k	W/(m·°C)
Thermal resistance	R	°C/W

2.6 Summary of Electrical Analogy

System	Current	Resistance	Potential Difference
Electrical	I	R	ΔV
Cartesian Conduction	q	$\frac{L}{kA}$	ΔT
Cylindrical Conduction	q	$\frac{\ln {r_2}/{r_1}}{2\pi kL}$	ΔT
Conduction through sphere	q	$\frac{1/r_1 - 1/r_2}{4\pi k}$	ΔT
Convection	q	$\frac{1}{h \cdot A_s}$	ΔT

Illustration 2.1

The two sides of a wall (2 mm thick, with a cross-sectional area of 0.2 m^2) are maintained at 30^oC and 90^oC. The thermal conductivity of the wall material is 1.28 $W/(m\cdot{}^oC)$. Find out the rate of heat transfer through the wall?



Solution 2.1

Assumptions

1. Steady-state one-dimensional conduction
2. Thermal conductivity is constant for the temperature range of interest
3. The heat loss through the edge side surface is insignificant
4. The layers are in perfect thermal contact

Given,

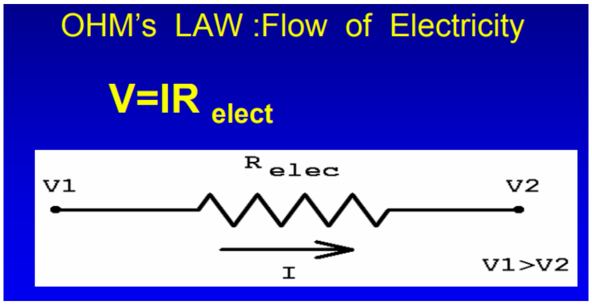
$k = 1.28\ W/(m\cdot{}^oC)$ $A = 0.2\ m^2$ $x = 2\ mm = 0.002\ m$

The rate of heat transfer can be written as,

$$\dot{q} = \frac{\Delta T}{x/_{kA}}$$

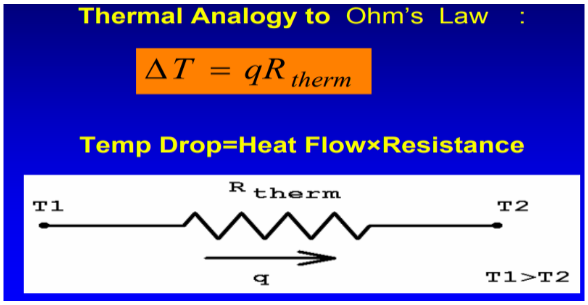
$$\dot{q} = \frac{90 - 30}{0.002/_{1.28\ \times 0.2}} = 7680\ W$$

Analogy between Flow of Electricity and heat transfer:



OHM's LAW :Flow of Electricity

$V = IR_{elect}$



Thermal Analogy to Ohm's Law :

$$\Delta T = qR_{therm}$$

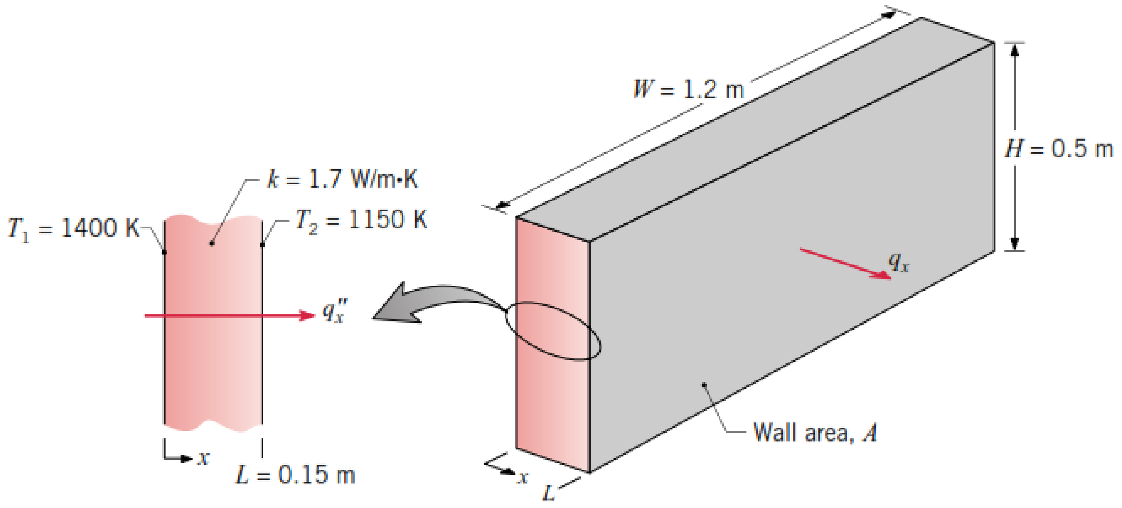
Temp Drop=Heat Flow×Resistance

EXAMPLE-2:

The wall of an industrial furnace is constructed from 0.15-m-thick fireclay brick having a thermal conductivity of 1.7 W/m.K. Measurements made during steady state operation reveal temperatures of 1400 and 1150 K at the inner and outer surfaces, respectively. What is the rate of heat loss through a wall that is 0.5 m by 1.2 m on a side?

Solution:

Known: Steady-state conditions with prescribed wall thickness, area, thermal conductivity, and surface temperatures.



Assumptions:

1. Steady-state conditions.
2. One-dimensional conduction through the wall.
3. Constant thermal conductivity.

Analysis: Since heat transfer through the wall is by conduction, the heat flux may be determined from Fourier's law.

$$q''_x = k\frac{\Delta T}{L} = 1.7 \text{ W/m} \cdot \text{K} \times \frac{250 \text{ K}}{0.15 \text{ m}} = 2833 \text{ W/m}^2$$

The heat flux represents the rate of heat transfer through a section of unit area, and it is uniform (invariant) across the surface of the wall. The heat loss through the wall of area is then

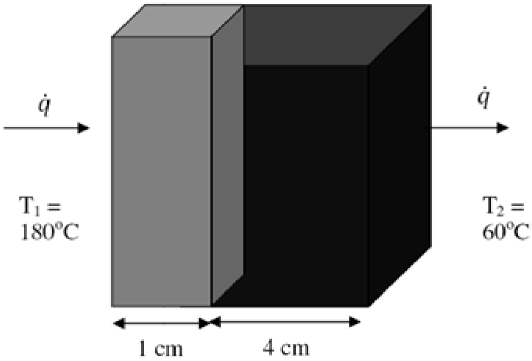
$$q_x = (HW)\,q''_x = (0.5 \text{ m} \times 1.2 \text{ m})\, 2833 \text{ W/m}^2 = 1700 \text{ W}$$

Comments: Note the direction of heat flow and the distinction between heat flux and heat rate.

Illustration 2.2

One side of a 1 cm thick stainless steel wall (k_1 = 19 W/m·°C) is maintained at 180°C and the other side is insulated with a layer of 4 cm fibreglass (k_2 = 0.04 W/m·°C). The outside of the fibreglass is maintained at 60°C and the heat loss through the wall is 300 W. Determine the area of the wall?

Solution 2.2



Assumptions:

1. Steady-state one-dimensional conduction.
2. Thermal conductivity is constant for the temperature range of interest.
3. The heat loss through the edge side surface is insignificant.
4. The layers are in perfect thermal contact.

Given, k_1 = 19 W/m·°C k_2 = 0.04 W/m·°C

x_1 = 1 cm = 0.01 m x_2 = 4 cm = 0.04 m

The resistance of the above composite,

$$R = \frac{x_1}{k_1 A} + \frac{x_2}{k_2 A}$$

Putting all the known values,

$$R = \frac{0.01}{19\,A} + \frac{0.04}{0.04\,A}$$

Thus,

$$\dot{q} = \frac{T_1 - T_2}{R}$$

$$300 = \frac{180 - 60}{\frac{0.01}{19\,A} + \frac{0.04}{0.04\,A}}$$

On solving, $A = 2.5\ m^2$

The previous discussion showed the resistances of different layers. Now to understand the concept of equivalent resistance, we will consider the geometry of a composite as shown in fig.

The wall is composed of seven different layers indicated by 1 to 7. The interface temperatures of the composite are T_1 to T_5 as shown in the fig.2.6a. The equivalent electrical circuit of the above composite is shown in the fig 2. below,

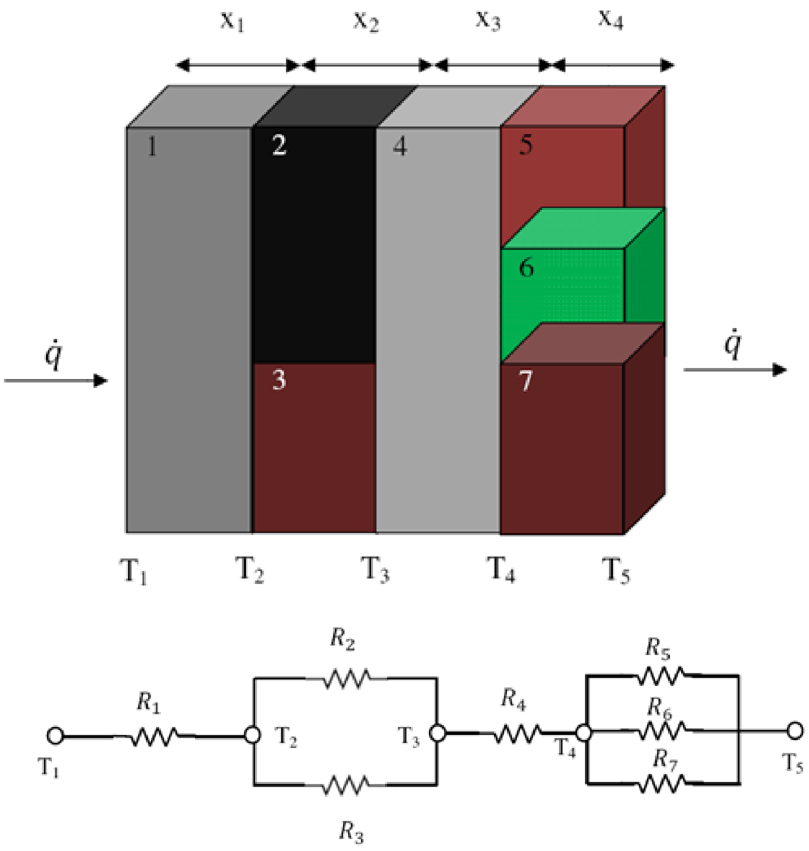


Fig. 2 Composite wall, and (b) equivalent electrical circuit

The equivalent resistance of the wall will be,

$$R = R_1 + \frac{1}{{}^1/_{R_2} + {}^1/_{R_3}} + R_4 + \frac{1}{{}^1/_{R_5} + {}^1/_{R_6} + {}^1/_{R_7}}$$

where,

$$R_i = \frac{x_i}{k_i\,A_i}$$

Therefore, at steady state the rate of heat transfer through the composite can be represented by,

$$\dot{q} = \frac{T_1 - T_5}{R}$$

where, R is the equivalent resistance.

Heat Transfer
Prof. Sunando Dasgupta
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 02
Introduction to Heat Transfer (Contd.)

So, by now we are familiar with the modes of Heat Transfer and the fundamental relation of conduction, which is Fourier's law, which gives you the heat flux vector as consisting of components in 3 directions and the components in each direction can be expressed as proportional to the temperature gradient in that direction.

In other words q_x, which is the x component of the heat flux, vector q is going to be proportional to $\delta T/\delta x$, which is the derivative and is the change in temperature in the x direction and the proportionality constant being a property of the material which is known as thermal conductivity. What we are going to do next is fundamental relation, which is known to all of us which is conservation of energy. So, what does conservation of energy tell us and how the conservation of energy can be related to heat transfer, in order with an aim to obtain that temperature at a specific point at a given instant of time, because in heat transfer that is what is required! You need to find out what is the temperature of an object at a specific point and at a specific instant of time.

Since, we are studying conduction right now we understand that conduction is essentially energy, which depends on temperature gradient. So, we need to precisely know what is the temperature profile inside an object, in order to find what is its gradient, which when multiplied with k would give us the flow of heat through a plane, which is perpendicular to the point where the temperatures are uniform.

So, if we have 2 points at 2 different temperatures this point will call it as an isothermal surface where everywhere the temperature is equal to T 1 and this is the T 2 another surface and the amount of transfer between these two would simply be governed by Fourier's law, but the concept of control volume is going to be important.

Because not only you are going to have transport of heat between point 1 and point 2 there will be situations in which you are going to have heat generation between point 1 and point 2 as well. So, let us think of a current flowing through an electric wire. So, we are going to have some sort of joule heating which is present in between two points that you need to take into account in order to find what is the temperature of the object at any given point?

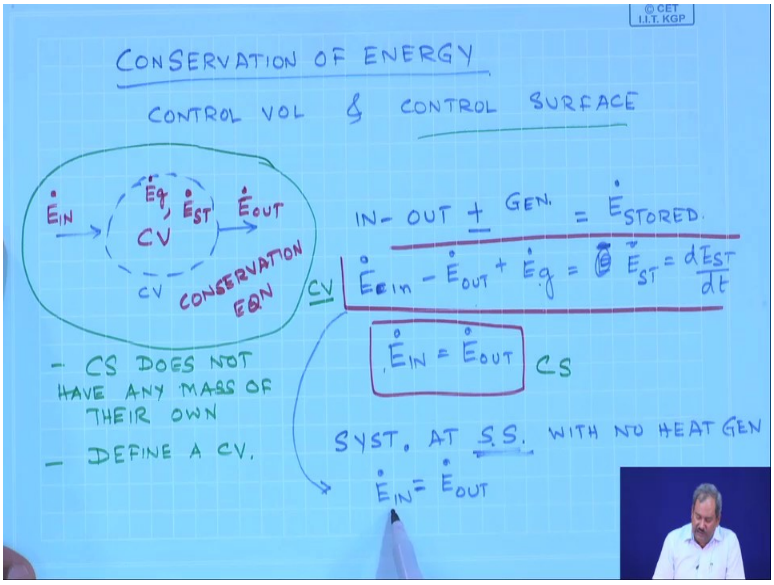
So, when you are generating heat and when you are having some amount of heat flowing in and some amount of heat flowing out. And as a result of all this the total energy content of the volume that you see will keep on changing. In other words let us think of just a block of copper, which you have placed on a hot surface and some amount of energy enters the block of copper, because of its proximity with a hot surface. And as it enters let us assume that no heat leaves from the copper surface to the surrounding. Let us say to begin with the temperature of the copper surface is the same as that of the surrounding, so, no heat transfer takes place. If it is different as the copper surface becomes more and more hot, it is going to lose energy from all the other surfaces to the surrounding.

So, you have some energy in by conduction, some energy which is going out of the copper block to the atmosphere again by conduction. And let us say by some means you are generating some amount of heat inside the inside the copper block and it is an unsteady state process.

So, if I think of an instant of time the amount of energy which comes in to the copper block, the amount of energy which goes out of the copper block, the amount of energy which you are generating artificially inside the copper block, all these would result in the total amount of energy which is stored in the copper block.

So, we are going to talk that is what conservation of energy is all about, the amount in minus the amount out plus any heat generation, that is taking place must be equal to the energy to the time rate of change of energy contained within the copper block. So, this copper block is therefore, is termed as the control volume. So, control volume is what in something it is an enclosed space, which has a specific mass and the form of conservation law, which is applicable for such a control volume. I am going to write that which is in minus out plus or minus, plus is for where energy is generated inside the control volume.

(Refer Slide Time: 05:35)



CONSERVATION OF ENERGY

CONTROL VOL & CONTROL SURFACE

IN − OUT ± GEN. = $\dot{E}_{STORED}$.

$$\dot{E}_{in} - \dot{E}_{OUT} + \dot{E}_g = \dot{E}_{ST} = \frac{dE_{ST}}{dt}$$

$$\boxed{\dot{E}_{IN} = \dot{E}_{OUT}}$$

- CS DOES NOT HAVE ANY MASS OF THEIR OWN
- DEFINE A CV.

SYST. AT S.S. WITH NO HEAT GEN

$$\dot{E}_{IN} = \dot{E}_{OUT}$$

And minus when there is some depletion of energy, which is getting depleted inside the control volume and will see the examples of that in this. So, this must be equal to stored, the time rate of change of energy stored in all these in and the out. So, this is

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \dot{E}_{st} = \frac{dE_{st}}{dt}$$

So, this is the conservation equation and the enclosed space with the fixed mass is known as the control volume. So, this is a control volume and you have some amount of energy, which is coming in and this is the time rate of energy, which is going in out and there may be some amount of energy, which is generated in here as a result of which you are going to have the rate of energy stored inside the control volume will also be different.

So, the conservation equation can be written for a control volume that has a fixed mass. So, this is what control volume is and this is the form of the conservation equation that you would expect you would expect for such a situation, then what is a control surface? Control surface is again an imaginary surface, which together form the boundary of the control volume. So, if you think of a copper block a rectangular copper block, then the 6 surfaces these and the 2 up and the top and the bottom, the 6 surfaces of the copper block those are known as the control surfaces, which defines the control volume, which defines what is the volume, what is the mass of copper, which is contained in the space. So, they are known as control surfaces. By definition control surfaces do not have any mass of their own whereas, control volumes do. So, the major distinction between control volume and control surface is control surfaces do not have any mass. So, that is the difference between control surface and control mass. So, the conservation equation, which we did right for the control volume has to be modified for control surfaces as they do not have any mass of their own. If they do not have any mass of their own, then they cannot store any energy and if there is no mass of the control surface then they also cannot generate any energy.

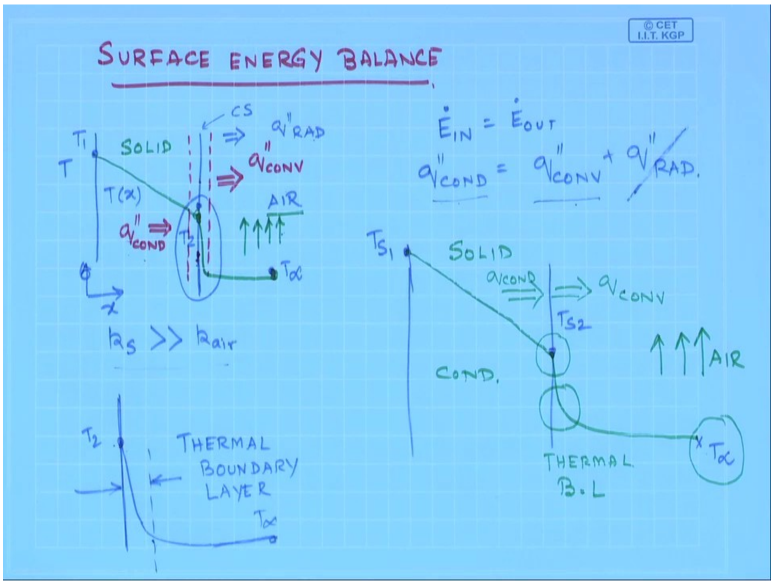
So, for a control surface in the conservation equation, you need to drop the $\dot{E}_g$ term, the time rate of change of energy generated in the system, as well as the $\dot{E}_{st}$ that is the time rate of change of energy stored in the control volume. So, for a control surface the conservation equation would simply turn out to be $\dot{E}_{in} = \dot{E}_{out}$. So, this is what you are going to get for a control surface. So, this is for a control surface and what you have here is for a control volume. So, that is the difference between a control volume and the control surface formulation as well as the conservation of energy is concerned.

So, this is primarily as a control surface (CS) as a CS does not have any mass of their own and they simply define a control volume. And these are the forms of the conservation equation. Now, if you look at this equation the conservation equation for a control volume there is what is going to be.It is what is going to how what it would look like if I apply it for a steady state process. What if there is no generation which is present in such a system? It is a case of the block again where 2 surfaces are maintained at 2 different temperatures, ok.

However, there is no generation of heat in there and you allow the process, allow the process sufficient time such that a steady state has reached. So, the moment a steady state is reached the amount of energy stored inside the control volume does not change, cannot change, ok. And, if there is no heat which is generated inside the control volume then that term will also disappear. So, what you have for a system at steady state, so, for if you have a system at steady state with no heat generation, this equation would simply be equal to $\dot{E}_{in} = \dot{E}_{out}$.

So, when you when you when you look at these 2 equations in absence of any heat generation and at steady state, this becomes equal and becomes identical with the control surface equation. However, fundamentally conceptually you must appreciate what is the difference between a control surface and a control volume. The next what I am going to write is surface energy balance.

(Refer Slide Time: 13:00)



So, let us say that let us assume that I have a block this is my x direction and the over here I have the temperature this is the temperature axis and this edge I am taking as my control surface. So, the temperature is T_1 here and the temperature is T_2 at this point and over here it is exposed to an ambient air ambiance with the temperature equal to T_∞ at this point.

So, between this is a solid and here you have as air. So, in the solid the principle mode of heat transfer in, since there cannot be any motion of the molecules. So, in a solid the heat gets transferred from 1 point to the other by means of conduction only, as the molecules cannot on an average move from their positions. So, in absence of any heat generation in a system and we will see subsequently why in absence of heat generation and at steady state the temperature distribution is going to be linear.

So, the temperature distribution in such a case will be linear, and therefore this is going to be the difference and this is going to be the profile where this T is a function of x, but it is a linear function of x. Now let us say that , this is my control surface. So, the red line simply denotes the extend denotes that this is my this is this is my control surface ok.

So, you are going to have q” conduction which is moving towards the control surface and over here from the control surface the heat is going towards air by q” convection. So, you have heat gets transported in the solid by conduction, heat gets transported in the air by convection and this is a reasonable approximation since we realize that thermal conductivity of the solid is much more than thermal conductivity of air.

So, the principle mode of heat transfer on the air side of this solid interface is nothing, but convection. So, you can also assume that the air is moving with certain velocity along the exposed sides of the block. Not only convection you may also have, if the temperature of the solid is large radiation which is going on.

So, this surface the control surface can lose heat as a result of convection as well as conduction as convection as well as radiation. So, the heat that travels to the control surface is; obviously, only by conduction and the heat which is lost from the surface is convection and radiation. And, since we know that the control surface has no mass of its own the conservation equation would simply be $\dot{E}_{in} = \dot{E}_{out}$.

And when you take this as the surface then q” conduction is simply going to be equal to q double prime convection plus q double prime radiation. So, most of the cases the radiation becomes important only when the temperature is quite large, in many realistic situations the radiation does not play a significant role as compared to conduction and convection.

So, when that happens we can also drop this radiation in here since it may not be that significant. So, we have conduction equality of conduction and convection across a control surface, 1 side of which faces the solid the other side is a flowing fluid of low thermal conductivity because this is a prerequisite low thermal conductivity and therefore, convection is the prevalent mode of heat transfer and your $q_{conduction}$ is equal to $q_{convection}$.

But, how does the temperature profile go from here to T_∞. And therein lies something which I will have to explain to you later. You will see more in greater detail that there exists something, which is called as a boundary layer or a thermal boundary layer. What is proposed is that all the temperature difference from T_2 to T_{∞}, this drop in temperature from T_2 to T_∞ takes place over a region, which is very close to the solid surface and then it asymptotically approaches the value of the temperature T_∞.

So, if I enlarge this section it would look something like this. So, this is my control surface this is T_2 and the temperature profile changes drastically now this is my T_∞. So, temperature profile changes drastically over a region, which is very thin and this region where the temperature changes from that of the base temperature to the temperature of the free stream to the temperature of the free flowing air which is T_∞, this is known as the thermal boundary layer.

So, in a thermal boundary layer as I have shown you the temperature drops suddenly the temperature changes suddenly till it reaches the temperature of the free stream. We will discuss more about why and what is the significance of thermal boundary layer, is there any experimental proof that there is something called a thermal boundary layer.

So, we would discuss that in subsequently, but what you have to accept now for the time being is that for any convection process, the temperature change is sharply near the solid fluid interface. And it reduces or increases depending on whatever be the case sharply enriches the value of the free stream temperature. And the thickness over which this change over takes place is commonly known as the thermal boundary layer. I think in your fluid mechanics you have you have already studied what is hydrodynamic boundary layer thickness.

So, if you have a flat plate and a liquid is flowing over it then due to no slip condition the velocity of the fluid on the solid liquid interface on the solid is going to be zero, that is the no slip condition, but if you go above slightly above the plate the velocity here is equal to the free stream velocity, which flows unperturbed over the solid surface if the distance from the solid surface is beyond certain value.

So, the region over which this change in velocity from 0 to the free stream velocity takes place is known as the hydrodynamic boundary layer. And you also probably know that the viscous forces are important inside the thin hydro dynamic boundary layer, outside of this boundary layer, the flow can be termed treated as inviscid or zero viscosity. So, you have also probably have heard about 2 equations one is Navier Stokes equation and the second is Euler’s equation.

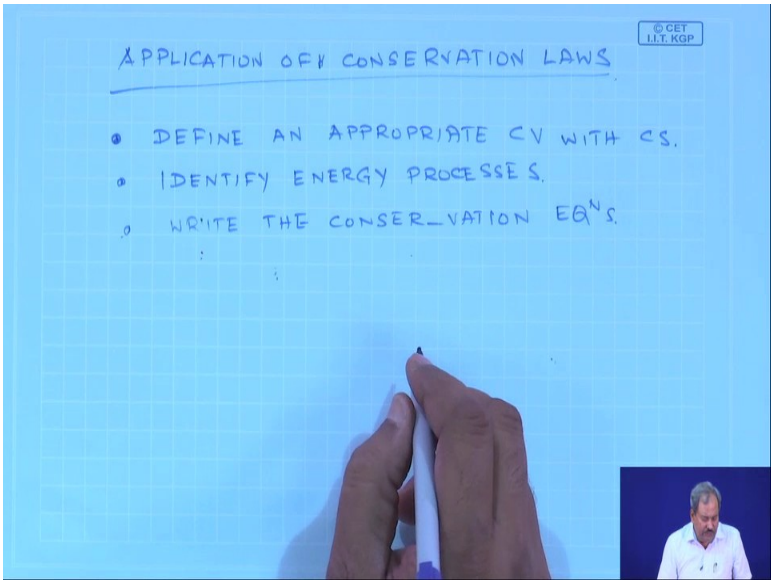
So, Navier Stokes equation takes into account the viscosity the viscous forces, which are present in a flowing fluid, whereas Euler's equation assumes that the flow is inviscid and the famous Bernoulli's equation can be derived from Euler's equation and in its true form the Bernoulli's equation can be used only for inviscid fluids. So, inside the the momentum boundary layer, inside the hydrodynamic boundary layer, viscous forces are present, outside of the boundary layer the viscous forces are unimportant, are not present.

So, this is that is the concept of hydrodynamic boundary layer. So, which I think you know. So, based on same concepts the concept of thermal boundary layer is also prescribed, where the temperature is going to be the same as that of the solid, when we think about the fluid very close to the solid surface, but as we move along the temperature changes sharply. And over a small thickness it varies from T_S the temperature of the solid to T_∞, which is the temperature of the fluid, which is flowing past the solid surface.

So, that is the thermal boundary layer, the important point here is what I need to show you is what would the profile the temperature profile look like for such a case. So, this is my solid and this is the temperature of the air and this side is maintained at a high temperature T_{S1} and at a temperature T_{S2} over here as there is no heat generation an it is a steady state case, it is going to be just a linear change from T_{S1} to T_{S2} inside the solid where only conduction is present, outside of which the air or the air is flowing with some velocity and a temperature constant temperature of T_∞ which is lower than T_{S2}.

So, the temperature change will be very sharp near the solid wall. So, this is what is called as the thermal boundary layer and the complete profile would look something like this. So, what you have here then is linear profile sharp change asymptotically approaching the value of T_∞. And, if you consider this point the flow due to conduction must be equal to the flow due to convection, of course, assuming that there is there is no radiation, radiation is not important in such a case. So, what is the methodology that one has to follow and what is the conservation methodology analysis of application of conservation laws?

(Refer Slide Time: 26:00)



APPLICATION OF CONSERVATION LAWS

- DEFINE AN APPROPRIATE CV WITH CS.
- IDENTIFY ENERGY PROCESSES.
- WRITE THE CONSER_VATION EQ^{N}S.

So, how do we apply the conservation laws? The first step is you define an appropriate control volume, ok. Appropriate control volume with control surfaces. Then the second is identify the energy processes and write the conservation equation.

So, these are the 3 steps which one should follow in order to use in order to apply the conservation laws. So, what you do then is first you identify the control volume and the control volume is defined in terms of control surfaces, you identify the energy processes is it a conduction, is it convection, is it both conduction and convection, whether radiation is important or not you make those judgements and then you write the conservation equations.

And we so far, we know only about what to write for conduction. We are not sure what to write for convection or for radiation, since we have not discussed that, but I will show you what what would be the form of the rate equations for that. So, identify whether it is conduction, convection or radiation and express the heat flow in using certain laws for example, Fourier's law for the case of heat conduction.

And, then what you are going to get is a governing equation for the process and I will show how this governing equation for any system can be derived, can be developed. One more point before we conclude is that the conservation equations is valid for at differential control volumes, as well as for an integral control volume.

So, we can assume that my control volume, its size is dx dy and dz. And then if we write it appropriately what you are going to get is a differential equation. And, if you can solve that differential equation based on an energy conservation, if you can solve that differential equation you get temperature as a function of space coordinates, that is x y and z and if it is an unsteady state process also as a function of time.

So, your temperature T as a function of x y z and time t is going to be the result of the differential equation, which you have obtained for a differential control volume and the approach by which the temperature at every point and at every instant, if it is, if it is possible to obtain that the approach that gives you that is known as differential approach.

There is there is another approach, which does not care about how temperature varies at every point in the in the control volume or at every instant. Though, in that approach we will be satisfied with how does the system behave in an overall sense to obtain what would be the overall behavior of this system as it gains energy, loses energy, and when it comes towards a steady state.

So, the conservation equations can also be applied to an integral volume, which unlike the differential approach has a finite length, height, and width. So, you have L_1 length, width and height as well as well as it is a function of time. So, the approach which will give you an overall approach, which is will give you an overall value of the temperature at a plane not at every point is known as the integral approach. For times you are more interested in how does the system behave generally rather than what is going to happen to each point in the system.

So, on one hand you have a detailed approach, which is a differential approach that we would see also you have an approach which can also be termed as an engineering approach, which is a control volume approach, where you are not interested in obtaining the specific value and average value would do, ok.

So, you do not want to know how temperature at every x varies, you just want to know what is the temperature of this plane the average temperature of this plane. So, the approach that gives you that is known as the control volume approach, the control volume approach, integral approach and you also have the differential approach, and we would see what those are so, but the bottom line is using your conservation equations somehow you have to write the governing equation for the system.

Once you have the governing equation you should be able to solve it utilizing some of the boundary conditions, because you know when you are integrating when you are solving a differential equation you end up with integration constants. These integration constants will have to be evaluated utilizing the physical description of the system. So, what is going to be those physical descriptions of the systems that will act as boundary conditions?

So, something has to be known for to convert the physical principles into boundary conditions. One example could be, and I will give you all the detailed examples in the next class, one example could be that the temperature at a specific point in your control volume is known to you.

So, let us talk about that you have this pen whose you would like to find out the heat transfer taking place in this pen, but let us say that this edge of the pen is firmly in contact with a solid surface which is maintained at a constant temperature of a 100°. So, what you can say then is at when x equals 0 the temperature is known let us say 100°, temperature is known. So, at a spatial coordinate that the specific value of temperature may be known to you and that can act as boundary can act as a boundary condition.

So, expressing the description of the problem, expressing the understanding of your problem in physical terms would give you boundary conditions, which you are going to need to solve the governing equation. So, what would we do in the next class is first identify the common boundary conditions that one would expect and then what is going to be the form of the boundary condition considering only conduction because in their initial part we will restrict ourselves to conduction.

So, we will write what is the differential equation? What is the governing equation the differential governing equation for conduction in a solid and what are the possible boundary conditions? So, with the help of the equation and the boundary conditions we should be able to obtain temperature profiles in different geometries under different conditions and they that would give rise to some very interesting and useful results which we will take up in the next class.

Heat Transfer
Prof. Sunando Dasgupta
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 05
One Dimensional Steady State Conduction

Previously, we have derived what is the conduction equation for Cartesian co-ordinate systems, cylindrical systems, and spherical systems. In the development of those equations, we have assumed the temperature can vary in all three possible directions and it can vary as a function of time as well. And there may be a situation in which some heat is going to be generated inside the control volume.

So, the equation that we have that is the generalized governing equation for conductive heat transfer in a solid medium. But, we are going to see some of the special cases which arise out of this equation which has practical applications; for example, in many of the situations we see that a plane wall has layers of insulation on it, to reduce the heat loss during winter or heat gain during the summer.

So, when a plane wall has several layers of insulation attached on one side of it and if we know the temperatures of each of the junctions between the insulator and the wall, then it is possible to predict what is the total heat loss from the wall to the outside at steady state conditions. Because, in practical applications we mostly are concerned with what is going to happen at steady state. So, it is important therefore, to simplify the heat diffusion equation for special conditions when you have only a plane wall and at steady state. We would also add another restriction that there is no heat generation in the control volume that we are considering. Again, going back to the example of heat loss from the walls of a room at steady state with insulations on the outside of the wall; obviously, there is no heat generation in the insulation.

We would also need to appreciate the fact that at steady state, no matter whatever be the thickness of the insulation or the material of the insulation; the same heat goes through all the layers. So, the heat rate is constant the amount of heat which will be lost by a hot room with layers of insulation on the outside is a fixed quantity. And secondly, when you think about a plane wall and insulation on top of it, then you do not have any variation in the heat transfer area. So, the same area is exposed to heat transfer from inside the room;

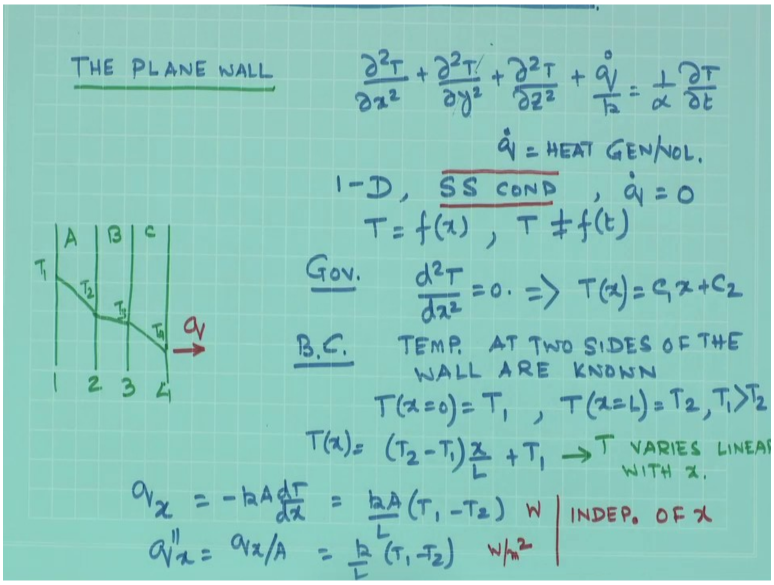
to the outside, and the cross-sectional area does not change. So, therefore, the special characteristics of a plane wall is that the area remains constant, and if the area remains constant we understand from physical principles; that since there is no heat generation and at steady state, the same amount of heat must travel through all these layers of insulation.

So, for planar systems the heat; both the heat rate and the heat flux are constant; which may not be the case for cylindrical systems or spherical systems. Because as you move away from the center of a cylindrical or a spherical surface, the radius increases and since the radius increases the area available for heat transfer also increases. So, the quantity of heat will remain constant, but the flux will not be constant. So, that is the major difference of conduction in planar systems, at steady state and conduction in radial or spherical systems.

The fact that heat flux may vary for non-planar systems will also lead to interesting observations, that it will also affect the temperature profile. In one case (for planar systems), the temperature distribution is going to be linear; whereas, in the case of cylindrical and spherical systems, they are going to be non-linear.

So, our starting point for analysis of conductive heat transfer in plane walls starts with the heat diffusion equation. And then we are going to simplify the equation based on its one-dimensional heat transfer; no generation of heat and at steady state.

(Refer Slide Time: 06:17)



THE PLANE WALL

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$\dot{q}$ = HEAT GEN/VOL.

1-D, SS COND, $\dot{q} = 0$

$T = f(x)$, $T \neq f(t)$

GOV. $\frac{d^2T}{dx^2} = 0. \Rightarrow T(x) = C_1 x + C_2$

B.C. TEMP. AT TWO SIDES OF THE WALL ARE KNOWN

$T(x=0) = T_1$, $T(x=L) = T_2$, $T_1 > T_2$

$T(x) = (T_2 - T_1)\frac{x}{L} + T_1 \rightarrow$ T VARIES LINEA WITH x.

$q_x = -kA\frac{dT}{dx} = \frac{kA}{L}(T_1 - T_2)$ W | INDEP. OF x

$q''_x = q_x/A = \frac{k}{L}(T_1 - T_2)$ W/m²

So, we start first with this equation which we have derived previously in the class:

$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{\dot{q}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$. So, this is temperature as a function of x, y, z, as well as time.

So, there is a transient component also attached to this and $\dot{q}$ is the heat generated per unit volume. So, therefore if it is 1 dimensional and steady state condition then I can say that T is a function only of x; let us assume that x is the direction in which the heat flow takes place and T is not a function of time.

So, when I talk about steady state what I mean is that the temperature can be a function of position, but at a given position the temperature is not a function of time. So, for the 1-dimensional case temperature is a function of x, but temperature is not a function of time and we will also assume that there is no heat generation in the system therefore $\dot{q}$, is 0. So, if we apply these 3 conditions to the above equation, the governing equation reduces from the heat diffusion equation as $\frac{\partial^2 T}{\partial x^2}=0$.

Integrating the above equation would simply give you the temperature as a function of x as: $T=C_1x+C_2$; where C_1 and C_2 are constants of integration. So, now to solve this I need boundary conditions. Specifically, two boundary conditions are required and what you would consider is that the temperatures at the two sides of the wall are known.

Let us consider that T at x = 0; that means, at one edge of the wall the temperature let us say it is a T_1 and T at the other edge of the wall which is at x = L is equal to T_2. And we will assume that T_1 is greater than T_2 though the derivation will work fine; even if you do it in the other way that T_2 is greater than T_1; only the direction of heat transfer will be different.

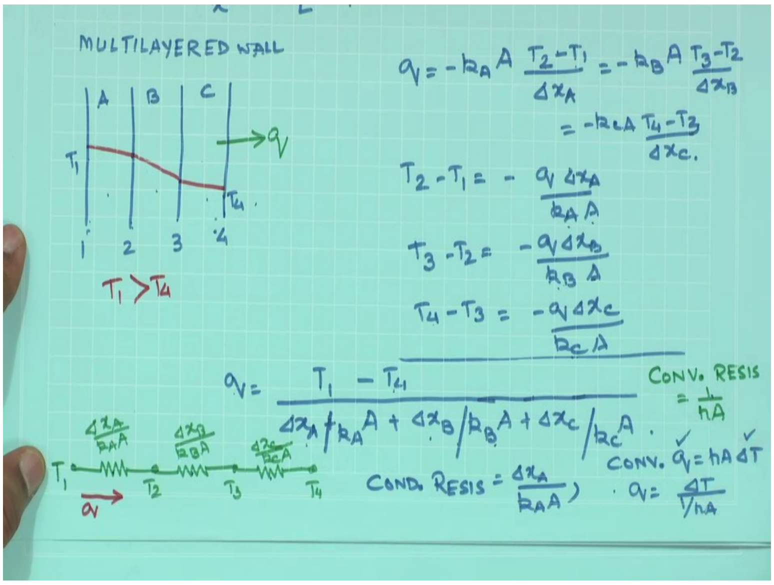
So, when you use these two boundary conditions, the solution would turn out to be:

$T(x) = (T_2 - T_1)\frac{x}{L} + T_1$. So, this tells me that temperature varies linearly with x. So, this clearly again shows that the temperature at any position is a function of x only and it depends on the imposed temperature difference which is T_1 minus T_2; it also depends on the geometric parameter, this L is the thickness of the wall, and the imposed temperatures.

From Fourier's law we know that q, the heat flow in the x direction is equal to minus k times A dT/ dx where k is the thermal conductivity. So, one can write looking at this expression and finding out $\frac{dT}{dx} = \frac{T_1 - T_2}{L}$ and the heat flux is simply $\frac{q_x}{A} = \frac{k}{L}(T_1 - T_2)$.

The important point to note here is that the heat flow; the total rate of heat flow with units of Watt and heat flux with units of Watt per meter square both are independent of x. So, our derivation simply shows what we have started with the basic premise that both the heat rate and heat flux are constant in planar systems; which would not be the case for a spherical or radial system, where the cross sectional area keeps on changing as you move in the r direction.

(Refer Slide Time: 12:18)



MULTILAYERED WALL

$$q = -k_A A \frac{T_2 - T_1}{\Delta x_A} = -k_B A \frac{T_3 - T_2}{\Delta x_B} = -k_C A \frac{T_4 - T_3}{\Delta x_C}.$$

$$T_2 - T_1 = -\frac{q\,\Delta x_A}{k_A A}$$

$$T_3 - T_2 = -\frac{q\,\Delta x_B}{k_B A}$$

$$T_4 - T_3 = -\frac{q\,\Delta x_C}{k_C A}$$

$$q = \frac{T_1 - T_4}{\Delta x_A/k_A A + \Delta x_B/k_B A + \Delta x_C/k_C A}.$$

CONV. RESIS $= \frac{1}{hA}$

CONV. $q = hA\,\Delta T$ ✓

$q = \frac{\Delta T}{1/hA}$

COND. RESIS $= \frac{\Delta x_A}{k_A A}$

Now let us assume that we have a system of a plane wall with three materials which are A, B and C. There would be four possible junctions between these three, materials (denoted by 1, 2, 3, and 4 respectively) and let us assume that the temperature profile is linearly decreasing. And as you can see that as you move from 1 towards 4 the temperature decreases, and we know that heat always flows in the direction of decreasing temperature.

So, at steady state the same q will flow through all the materials - A, B and C. So, in a multi layered since q is constant we can write: $q_A = -k_A A_A \left(\frac{T_2 - T_1}{\Delta x_A} \right)$ for the first material; and $q_B = -k_B A_B \left(\frac{T_3 - T_2}{\Delta x_B} \right)$ for the second material and finally, for the third material - $q_C = -k_C A_C \left(\frac{T_4 - T_3}{\Delta x_C} \right)$.

Rearranging the above equations, we get:

$$T_2 - T_1 = -\frac{q_A \Delta x_A}{A_A k_A}$$

$$T_3 - T_2 = -\frac{q_B \Delta x_B}{A_B k_B}$$

$$T_4 - T_3 = -\frac{q_C \Delta x_C}{A_C k_C}$$

The k_A, k_B, and k_C are the conductive thermal conductivities of material A, material B, and material C respectively; and Δx_A, Δx_B, and Δx_C are the thicknesses of each of these materials.

Adding the above three equations and rearranging them by taking q as a common parameter would result in:

$$q = \frac{T_4 - T_1}{\frac{\Delta x_A}{k_A A_A} + \frac{\Delta x_B}{k_B A_B} + \frac{\Delta x_C}{k_C A_C}}$$

It is to be noted for the case of materials with same cross-sectional area, the area term in the denominator could also be taken to be a constant value.

So, what I can think of this as T 1 is one of my potential and let us say I have a resistance, then I have T 2 another resistance and I have T 3 and another resistance, and I have T 4.

So, temperatures can be viewed as potentials and the heat transfer resistances are: $\frac{\Delta x_A}{k_A A_A}$; $\frac{\Delta x_B}{k_B A_B}$; and $\frac{\Delta x_C}{k_C A_C}$. So, what you have then is the overall potential difference divided by the sum of resistance. So, only thing is the potential difference to be replaced in Ohm's law to arrive at the heat transfer law is overall temperature difference by sum of all conductive resistances.

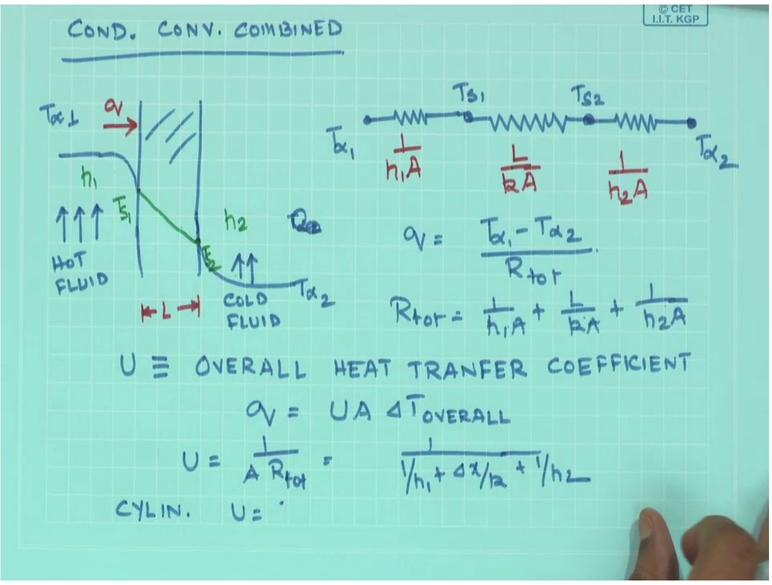
So, you have 3 or 4 materials sandwiched one after the other and if do know what the temperatures at the 2 end points are, then you can find out what is the flow of heat through this combination materials; through this multilayered wall. And the result that you would get is something like what you have obtained in the case of Ohm's law.

One thing to be noted here is that the conduction resistance is $\frac{\Delta x_A}{k_A A_A}$, whereas the convection resistance (from Newton's law of cooling) is equal to 1/h A. So, if a wall is connected with the convection environment then I am going to have 3 conduction resistances in series and a convection resistance being added to it.

We would now go to a situation which is more practical, where these walls through which conduction takes place they are exposed to an environment outside air. So, the room that I am sitting in it is an air-conditioned room; it has multiple layers of insulation on the outside and then it is exposed to the outside ambient temperature. So, the amount of heat loss from this wall is going to be combination of all the conduction resistances due to the insulation and the wall that we have for this room; as well as what is the convection resistance on the outside of the building which connects it to the ambient temperature.

We will quickly see how the result would look like for a system in which I have both conduction and convection; however, it is a planar system such that both the heat rate and the heat flux are going to be constant.

(Refer Slide Time: 23:02)



COND. CONV. COMBINED

$$q = \frac{T_{\infty 1} - T_{\infty 2}}{R_{tot}}$$

$$R_{tot} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A}$$

U ≡ OVERALL HEAT TRANFER COEFFICIENT

$$q = UA\,\Delta T_{OVERALL}$$

$$U = \frac{1}{A R_{tot}} = \frac{1}{1/h_1 + \Delta x/k + 1/h_2}$$

CYLIN. U =

So, I have conduction, convection both are combined, and the diagram probably looks something like this, where this side is T infinity 1 and this side let us say the temperature is T infinity 2.

So, I have a hot fluid which is moving up in this direction on this side of the wall and I have a cold fluid which is moving on the outside of this; so, this my wall. And the temperature here is T infinity 1, the temperature here is T infinity 2 and the temperature is going to sharply fall because of the thermal boundary layer that we have discussed before.

So, let us say this is the surface temperature of 1 and then I have a T S 2 which is the surface temperature of 2 and then it falls to T infinity 2. So, over here the fall is also going to be like this and in between T S 1 and T S 2; since it is a plane wall, at steady state without any heat generation; the temperature profile must be linear. The temperature T infinity 1 is more than T infinity 2 and therefore, the flow of heat is going to be from the left to the right.

So, this is a system in which I have conduction in the plane wall, convection on the inside as well as on the outside of the plane wall. So, if I draw the resistance diagram for this where the outside potential is T infinity 1 and the potential over here is T infinity 2; these are the 2 temperatures then I have some sort of a convection resistance which brings me to T S 1 and then the conduction resistance which brings me to T S 2 as per the diagram and then I have a convection resistance which then results in asymptotically and smoothly merges with T infinity 2.

So, this is my convection resistance which must be equal to and let us assume that this hot fluid maintains a convection coefficient h1 over here and a convection coefficient of h2 at this point. So, therefore, the resistance is simply going to be $\frac{1}{h_{1A}}$; as we have discussed before. The conduction resistance is going to be $\frac{L}{kA}$; where L is the thickness of the wall, A is the area, k is the thermal conductivity and over here it is going to be $\frac{1}{h_{2A}}$.

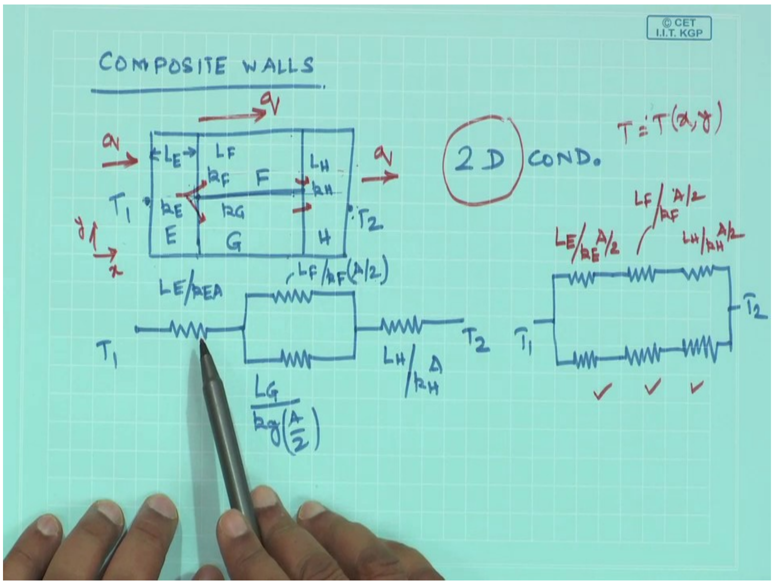
So, that is what we would get in this case therefore, the total flow of heat q is simply going to be T infinity 1 minus T infinity 2 divided by R total; where R total refers to the algebraic sum of these 3 resistances; since they are in series. So, this is R total would simply be equal to $\frac{1}{h_{1A}}+\frac{L}{kA}+\frac{1}{h_{2A}}$ that is the total resistance to heat transfer in this system.

It is sometimes advisable to express these 3 resistances is in terms of an overall heat transfer coefficient. And we would see the reference to overall heat transfer coefficient coming many times in convective heat transfer not only in convective heat transfer, but more commonly in the case of designing of heat exchange equipments, where instead of each of these individual resistances we express them in terms of an overall resistance and this is generally denoted by the symbol U and therefore, the q the heat flow is going to be U A, delta T overall as I mentioned the U appears mostly in the case of conduct convective heat transfer.

So, the total amount of heat transfer is simply going to be according to Newton's law of cooling as q equals U A delta T, where U is the overall coefficient which may contain both conduction and convection as we have seen in the example that we are discussing. The A is the cross-sectional area through which the heat transfer takes place and delta T is the overall temperature difference.

And it; obviously, does not assume that it is a linear or anything, but we understand that it is going to be linear in the solid, but it is going to have a shape which is where the temperature from the ambient comes to the temperature of the surface in a fashion which is consistent with our explanation based on thermal boundary layers.

(Refer Slide Time: 30:34)



Now, let us look at composite walls which are slightly different. Let us say you have a system in which up to this point, the wall is made of some material and from this point onwards it is of another material, but in between you have 2 different materials.

So, you have a material E with a thermal conductivity k E of length L E; for this one its length is going to be LF, the thermal conductivity is going to be the k F and this is material F. And I have k G in here, this is material G and this is material H where it is going to be L H and k H and the temperature here on this side is going to be the T 1 on this side it is going to be T 2.

So, as you can see that the heat is going to flow; if T 1 is greater than T 2 heat is going to flow from 1 to 2, but in here after it reaches this surface; part of the heat is going to move through this, part of the heat is going to move through this and they are going to emerge in material H and flow. So, this is the overall direction of q, but at this location at this plane, it is going to get divided and at this plane it going to combine; so that you get the same q and here you also going to get q.

This is a 2 D condition where the temperature is not a going to be a function of x alone, it could also be a function of y. So, there are 2 ways by which you can represent this in terms of an equivalent electric circuit. So, you have T 1 on one side T 2 on the other side and here the resistance is simply going to be $\frac{L_E}{k_E A}$ and here it is going to be $\frac{L_F}{k_F \frac{A}{2}}$. We write A/2 because we assume that half of the area is taken up by the material F and half of the area is taken by material G.

So, that is why I am putting it A by 2; it can be any fraction and it does not matter one third of the area can be made of F and 2 third will be made of G. So, appropriately the fraction of area is to be is to be replaced over here and, in this case, it is going to be $\frac{L_G}{k_G \frac{A}{2}}$; as I said A by 2 is just for this specific case. And over here it is going to be $\frac{L_H}{k_H A}$.

So, you have 3 resistances in series for the top part and so, this is my T 2 and this is my T 1 and heat is going to flow through this and through this. So, q 1 is flowing through this and q 2 is flowing through this where the sum of q 1 plus q 2 is going to be equal to q.

So, whenever you come across a composite walls; it is customary to express them either like this or in either like this or in this fashion, but in both cases we have to appreciate that near the junctions there would be 2 dimensional effects, 2 dimensional conduction where T is going to can be a function of both x and y where y is this direction and x is at this direction.

Explaining these complicated flows of heat through composite walls by a simple resistance in series or resistance in parallel mechanism would only give you an approximate value of the temperature, the junction temperatures and the temperatures the total flow of heat. So, this is an approximation and higher the difference between the thermal conductivities between the adjacent layers the error is going to be enhanced.

So, one should be careful about expressing the results in this specific form and most of the time, it is going to be dictated by how fast you want a result; whether you can tolerate approximations and what is the physical condition of difference in thermal conductivities such that the difference in temperature across a fixed x is not that significant. So, that must be considered before you start solving it.

Heat Transfer

Prof. Sunando Dasgupta
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 06
Temperature Distribution in Radial Systems

In the last class we discussed about what would be the resistance for heat flow through a planar wall or a combination of several plane walls. And, there we saw that the temperature difference between two ends of the wall can be viewed as the potential difference which causes the heat to flow from high temperature to low temperature and the resistance, if the heat flow can be expressed in the form which is similar to that of ohm's law where the potential difference is to be replaced by the temperature difference and the current is to be replaced by the heat flow. And whatever we have in the denominator of q equals δT by r, this r is equivalent to thermal resistance.

And the thermal resistance we have evaluated for a plane wall. And also if we have several such walls sandwiched one after the other then the overall temperature difference between the outer edge of wall 1 and the outer edge of wall n, then, this temperature difference is the overall temperature difference i.e. overall potential gradient. And the denominator is going to be a sum of all the individual resistances of each of these walls.

So, in essence the heat flow through a composite wall where cross sectional areas remain the same at steady state can be expressed as heat flow equal to the overall temperature difference by the sum of all resistances. And therefore, all these resistances it has been shown that they are in series.

So, based on that we now clearly understood that the resistance of the plane wall is going to depend on the geometry; that means, what is a cross sectional area, what is the thickness and will also going to depend on the thermo physical property which is the thermal conductivity of the specific wall that we are considering. So, the resistance is simply going to be L, where L is the thickness divided by k times A, where, k is the thermal conductivity of the material and A is the area which is perpendicular to the direction of heat flow.

Resistance = $\frac{L}{k.A}$

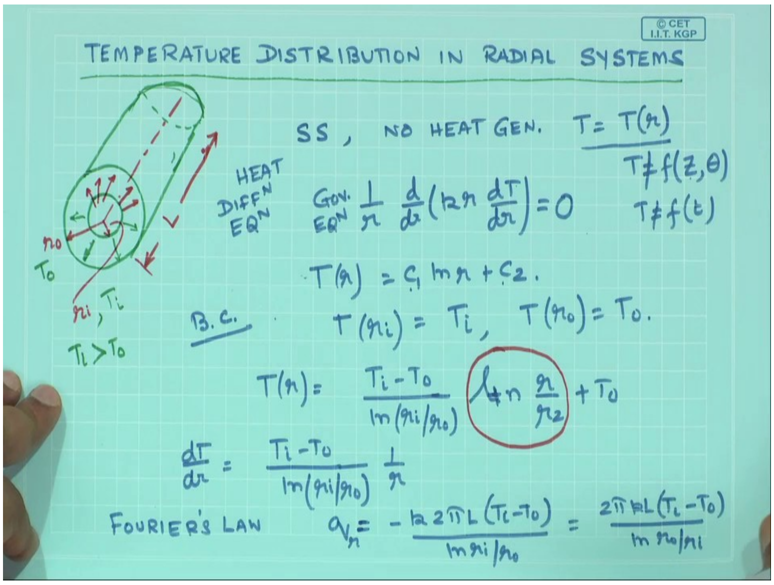
We have also seen that for composite walls of different type in which you have wall number 1 whose area is , then another wall of different thermal conductivity whose area is let us say A by 2 and another wall, wall number 3 whose area is a by 2, but it is of another material. So when such walls, composite walls of unequal cross sectional area are put together then there are two ways by which you can represent the heat flow; one is you can assume that there is going to be a series, that is the top 3 are going to be in series whereas the top and the bottom series are going to in parallel. So, we discussed about that as well.

One of the characteristic features of heat flow through plane wall is that the heat flow is constant, not only heat flow at steady state is constant, the heat flux is also constant since the cross sectional area which is perpendicular to the direction of heat flow does not change. So, as in planar systems the cross sectional area does not change with x, with the direction of heat flow therefore, both the heat rate and the heat flux are going to be constant.

However, I have mentioned in the previous class that for radial systems and for spherical systems in which as you move in the direction of r, the cross sectional area keeps on changing. And therefore, even though the conservation equation demands that heat rate has to be a constant, the flux may not be a constant. And therefore, we are going to have a different expression for the resistance to heat flow for the case of radial systems and for spherical systems. So, that is what we are going to analyze in today's class.

So, it is going to be radial temperature distribution at steady state and the mode of heat transfer is conduction, but you can always have convection at the outer or the inner edges of a radial of a wall. And therefore, it in that case it is going to become a convection conduction problem, but for the time being let us start first with conduction only case in a radial system.

(Refer Slide Time: 05:35)



TEMPERATURE DISTRIBUTION IN RADIAL SYSTEMS

SS, NO HEAT GEN. $\underline{T = T(r)}$

$T \neq f(z, \theta)$

$T \neq f(t)$

HEAT DIFF^N EQ^N GOV. EQ^N $\frac{1}{r}\frac{d}{dr}\left(kr\frac{dT}{dr}\right) = 0$

$T(r) = C_1 \ln r + C_2.$

B.C. $T(r_i) = T_i, \quad T(r_o) = T_o.$

$T(r) = \frac{T_i - T_o}{\ln(r_i/r_o)} \boxed{\ln \frac{r}{r_2}} + T_o$

$\frac{dT}{dr} = \frac{T_i - T_o}{\ln(r_i/r_o)} \frac{1}{r}$

FOURIER'S LAW $q_r = -\frac{k 2\pi L (T_i - T_o)}{\ln r_i/r_o} = \frac{2\pi k L (T_i - T_o)}{\ln r_o/r_i}$

So, the figure that you see over here is that of a that of a hollow cylinder whose inner radius is equal to r_i and the outer radius is r_0 and the temperature over here in at the inside is maintained at T_i whereas, the temperature at the outside is maintained at T_o. So, this is a situation the length of the hollow cylinder is L and we are trying to find out what is the flow of heat through this area to outside. So, if we assume that T_i is greater than T_o then this is the radial direction in which heat will flow through the solid material of the cylinder and we are trying to find out what is going to be the resistance for heat flow.

For this case for steady state condition with no heat generation and the case where T is as you can see is going to be a function of r only it is definitely not a function of θ and it is not a function of z. So, T is a function of r only therefore, the heat diffusion equation for radial systems reduces to only this much, because we do not have to consider the variation of temperature with r or θ. So, T is not a function of either z or θ. So therefore, all the terms containing gradient of T with respect to z or with respect to θ can be cancelled as it is a steady state one. So, T is not a function of time and there is no heat generation. So, therefore, if you look at the heat diffusion equation this is the form it is going to take ok.

$$\frac{1}{r}\frac{d}{dr}\left(kr\frac{dT}{dr}\right) = 0$$

And one can integrate this equation and what you get is

$$T(r) = C_1 \ln r + C_2$$

Boundary conditions:

$T(r_i) = T_i$; $T(r_0) = T_o$

So, this is your governing equation and in order to solve the governing equation you need 2 boundary conditions which are the 2 known temperatures in this case. So, T at r_i is equal to T_i and the second is T at r_0 is equal to T_o. So, when you put these 2 boundary conditions to evaluate C_1 and C_2, the final expression, that you are going to get is

$$T(r) = \frac{T_i - T_o}{\ln \frac{r_i}{r_o}} \ln \frac{r}{r_2} + T_o$$

So here, what I think we should appreciate is that the temperature distribution unlike the case of planar system is not a linear function of position, but it is going to be a logarithmic function of the radial location of the plate.

So, that is very important to note that it is no longer going to be linear distribution, it is going to be logarithmic distribution, and as you move in the direction towards the outer radius the area of heat flow, the area available for heat flow continuously increases. So, since it increases the temperature, it is not going to be linear and temperature here you can see it is going to be linear function of position.

Now, what is remaining is since we are trying to correlate cause and effect, the cause being the temperature difference and the effect being the heat flow and we are trying to find a relation or express our results in the form of something similar to ohm's law between cause and effect. So, if you do that to this equation then automatically the resistance to conductive heat transfer in radial systems will come out. So, that would be our next exercise to see what form the resistance would take in a radial system.

So, if you look at this equation once again then I can write this as

$$\frac{dT}{dr} = \frac{T_i - T_o}{\ln \frac{r_i}{r_o}} \frac{1}{r}$$

Using Fourier's law,

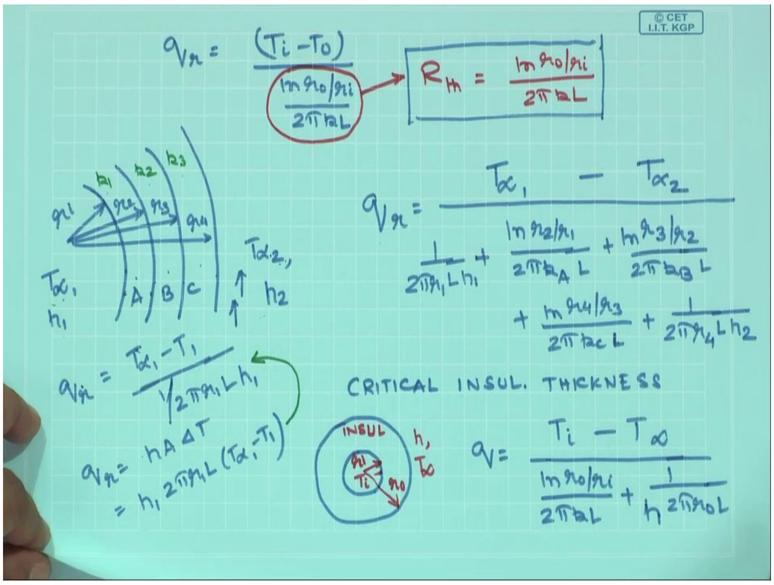
$$q_r = \frac{-k2\pi L(T_i - T_o)}{\ln \frac{r_i}{r_o}} = \frac{2\pi kL(T_i - T_o)}{\ln \frac{r_o}{r_i}}$$

So, if I take this to the next logical step and here you see this is the effect and this is the cause. So, if I bring twice by k L in the denominator what you have is something similar to ohms law

$$q_r = \frac{(T_i - T_o)}{\frac{\ln \frac{r_o}{r_i}}{2\pi kL}}$$

where the cause is the potential difference is a thermal potential difference which is expressed as temperature difference, and the effect is the radial flow of heat. And what I have in the denominator is $\frac{\ln \frac{r_o}{r_i}}{2\pi kL}$ that simply is the resistance to heat transfer for radial systems.

(Refer Slide Time: 12:15)



$$q_r = \frac{(T_i - T_o)}{\boxed{\frac{\ln r_o/r_i}{2\pi kL}}} \rightarrow \boxed{R_{th} = \frac{\ln r_o/r_i}{2\pi kL}}$$

$$q_r = \frac{T_{\infty 1} - T_{\infty 2}}{\frac{1}{2\pi r_1 L h_1} + \frac{\ln r_2/r_1}{2\pi k_A L} + \frac{\ln r_3/r_2}{2\pi k_B L} + \frac{\ln r_4/r_3}{2\pi k_C L} + \frac{1}{2\pi r_4 L h_2}}$$

$$q_r = \frac{T_{\infty 1} - T_1}{1/2\pi r_1 L h_1}$$

$$q_r = hA\Delta T = h_1 2\pi r_1 L (T_{\infty 1} - T_1)$$

CRITICAL INSUL. THICKNESS

$$q = \frac{T_i - T_\infty}{\frac{\ln r_o/r_i}{2\pi kL} + \frac{1}{h 2\pi r_o L}}$$

So, I am going to write in the next page as temperature difference. So, what you have here in the denominator is your resistance the thermal resistance which; obviously, is this.

$$R_{th} = \frac{\ln \frac{r_o}{r_i}}{2\pi k L}$$

So, this is an important result which gives us what would be the resistance to heat transfer in such case. Let us assume that you have a composite situation in which you have a number of walls like this, so this is r_2. Similarly have r_3 and r_4 with each one having thermal conductivity of k_1, k_2 and k_3 and you also have convection inside as well as convection outside, with the inside condition given as the temperature of the fluid in here is $T_{\infty 1}$ which is creating a convection coefficient of h_1 at the inner surface of the composite wall.

On the other side, you have $T_{\infty 2}$ as the temperature of the fluid which is made to flow on the outside which is also producing heat thermal convective heat transfer coefficient of h_2. This is quite common in many of the practical situations for example, in heat exchangers which we would see, where the hot fluid let us assume that it is flowing through a tube and somehow it is going to come in thermal contact with a cold fluid with which it will exchange heat. And therefore, the cold heat is going to be heated, going to absorb, going to gain energy out of this hot fluid.

And let us see hot fluid, fluid is flowing from point a to point b from where it is produced to where let us say it is going to a reactor. So, in the transit you want to maintain the heat loss to a minimum you want to keep the heat loss to a minimum. So, how do you do that, you put insulation on top of the pipe top of that tube. So, this insulation the pipe and the insulation they are may be 2 or 3 different types of insulation, one is an insulation which is thermally which is going to protect heat loss which is going to minimize heat lost and in order to protect the insulation you may have another outer cover.

So, as the heat travels from the inside to the outside it is going to experience different materials as it travels, and it is going to also see materials of different thermal conductivity. And as it travels it will see that the heat transfer area will keep on changing. So, under such circumstances what you are going to get is the picture that I have drawn where you have a composite wall where at the in the inside you have some convection coefficient. And at the outside you have a different convection coefficient which is probably provided by the ambient air which flows over the over the lagged pipe.

So, what would happen to such a case is in is depicted over here and I am simply going to write what is the form of the final equation going to be.

$$q_r = \frac{T_{\infty 1} - T_1}{\dfrac{\ln\frac{r_2}{r_1}}{2\pi k_A L} + \dfrac{\ln\frac{r_3}{r_2}}{2\pi k_B L} + \dfrac{\ln\frac{r_4}{r_3}}{2\pi k_C L}}$$

So, the radial flow of heat is going to be due to the temperature difference, the temperature difference inside minus temperature difference outside.

So, these are the 3 conduction resistance through the material A B and C, I also have a convection which is taking place over here. So, what is the convection the convective heat transfer will be

$$q_r = \frac{T_{\infty 1} - T_1}{\frac{1}{2}\pi r_1 L h_1}$$

This comes from Newton's law of cooling which simply says that

$$q_r = hA\Delta T = h_1 2\pi r_1 L_1 \left(T_{\infty 1} - T_1\right)$$

So, from here I have written it in the form of temperature difference by some sort of a resistance. So, this is the expression for convective heat transfer resistance in the case of radial systems. So, this is must be added to this overall resistance. Hence,

$$q_r = \frac{T_{\infty 1} - T_1}{\dfrac{1}{2\pi r_1 L h_1} + \dfrac{\ln\frac{r_2}{r_1}}{2\pi k_A L} + \dfrac{\ln\frac{r_3}{r_2}}{2\pi k_B L} + \dfrac{\ln\frac{r_4}{r_3}}{2\pi k_C L} + \dfrac{1}{2\pi r_4 L h_2}}$$

So, I think it is clear to you now is that what happens in this case where you have multiple walls radial walls one after the other of different thermal conductivities, the inside is exposed to a convection environment the outside is also exposed to another convection environment with 2 different temperatures and 2 different heat transfer coefficients.

So, the heat flow which has to remain constant in order to maintain equation of conservation of energy is the overall temperature difference which is $T_{\infty 1} - T_{\infty 2}$ by the sum of all resistances and by that I mean by the sum of all resistances is the convection resistance on the inside, all the possible conduction resistances that the heat flow is going to face. So, if there are 3 walls there is going to be 3 different conduction resistances and when it reaches the outer edge of the outer wall it experiences a convection once again. And from Newton's law of cooling we have seen how we can express the heat flow as temperature difference divided by a resistance and the convective heat transfer resistance is expressed as $\frac{1}{hA}$. And the area simply is going to be $2\pi rL$ that is the cross section that is the inner area of the inner cylinder.

So, you this way you can quite easily write what would be the form of the equation for radial flow in systems and as I mentioned they are very common in many industrial situations. Now what I would do next is something very interesting is normally what we feel is that, if I am feeling cold I just wear a sweater which is nothing but an insulation and this reduces the loss of heat from my body to the ambient and I do not feel cold anymore.

So, in the way the purpose of the sweater or the jacket is to ensure that the heat flow from my body to the ambient get slowered that is what is, that is what insulations do, they will use the flow of heat from the hot object to the cold object, but is it ever possible that by adding an insulation you are making more or higher flow of heat from the hot object to the cold object.

So, this is counter intuitive, but it may happen and I am going to show how under what conditions by adding insulations you simply increase the rate of flow of heat through that insulation. So, that is a very interesting concept it is known as the critical thickness of insulation. So, if your thickness of insulation is below a certain limit then by adding insulation you increase heat flow, if it is above the critical insulation thickness by adding insulation what you get is what we commonly expect that the heat flow rate reduces, but the interesting part is that region in which your sizes are is your size is below that of critical thickness of insulation so, by adding insulation you are increasing the loss of heat.

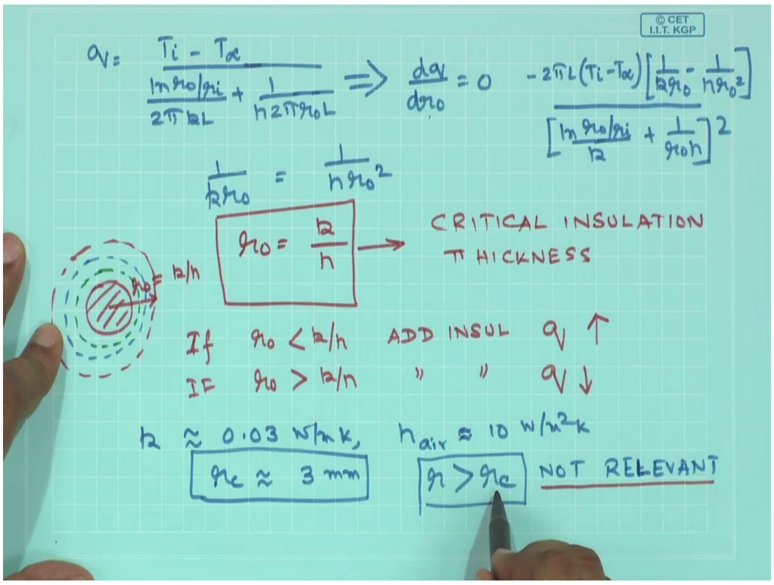
So, you would first mathematically see what needs to happen for you to decide; what is this critical thickness of insulation. And then we will talk about some of the practical uses, practical situations in which you may expect to encounter critical thickness of insulation and it has certain advantages as well and what is the scientific reason for such thing to happen. So, let us look at the derivation once again. So, next wego to critical thickness of insulation. Let us say I have a cylinder and a solid cylinder and an insulation. So, this is r_i and let us assume that the temperature here is maintained at T_i, temperature at the junction between the solid rod and the insulation. So, this is your insulation and the insulation radius is r_0 and on the outside it is exposed to a convection environment with temperature and heat transfer coefficient as h and T_∞.

So, it is a solid rod of radius r_i, the junction temperature is at T_i, the insulation radius is r_0 and it is exposed to h and T_∞. So, as before from here I can write that the heat flow is simply going to be

$$q_r = \frac{T_i - T_\infty}{\frac{\ln\frac{r_o}{r_i}}{2\pi k L} + \frac{1}{h 2\pi r_o L}}$$

So, there is nothing new I have simply used this for a system in for this case. So, if this is q then I am going to see is it possible to mathematically get at which point my heat flow is going to be maximum. So, if I can do that then I will probably get an idea of what is the concept of critical thickness of insulation.

(Refer Slide Time: 25:49)



© CET I.I.T. KGP

$$q = \frac{T_i - T_\infty}{\frac{\ln r_o/r_i}{2\pi kL} + \frac{1}{h2\pi r_o L}} \Rightarrow \frac{dq}{dr_o} = 0 \quad \frac{-2\pi L(T_i - T_\infty)\left[\frac{1}{kr_o} - \frac{1}{hr_o^2}\right]}{\left[\frac{\ln r_o/r_i}{k} + \frac{1}{r_o h}\right]^2}$$

$$\frac{1}{kr_o} = \frac{1}{hr_o^2}$$

$$\boxed{r_o = \frac{k}{h}} \rightarrow$$ CRITICAL INSULATION THICKNESS

$r_o = k/h$

If $r_o < k/h$ ADD INSUL $q \uparrow$

IF $r_o > k/h$ " " $q \downarrow$

$k \approx 0.03$ W/mK, $h_{air} \approx 10$ W/m²K

$\boxed{r_c \approx 3 \text{ mm}}$ $\boxed{r > r_c}$ NOT RELEVANT

So, starting with this

$$q_r = \frac{T_i - T_\infty}{\frac{\ln\frac{r_o}{r_i}}{2\pi kL} + \frac{1}{h2\pi r_o L}}$$

I am going to differentiate this and set this to be equal to 0. So, I am trying to find out is there a radius r_0 where r_0 is the insulation thickness which would maximize the flow of heat which is denoted by q.

$$\frac{dq}{dr_o} = 0 = \frac{-2\pi L(T_i - T_\infty)\left(\frac{1}{kr_o} - \frac{1}{hr_o^2}\right)}{\left(\frac{\ln\frac{r_o}{r_i}}{k} + \frac{1}{r_o h}\right)^2}$$

$$\frac{1}{kr_o} = \frac{1}{hr_o^2}$$

$$r_o = \frac{k}{h}$$

This r_0 is the critical thickness of the insulation which would maximize heat flow is known as the critical insulation thickness.

So, this critical insulation thickness therefore, it tells you that if you have a solid cylinder and you are putting insulations around it. So, this is your first level of insulation this is a and then at the next instant you increase the thickness of insulation some more and you are going to increase the thickness even more as long as your radius of this as you are adding. So, if this let say this is r_0 which is equal to k/h as long as you are below this value you are less than k_0/h your q will increase. So, if r_0 is less than k/h, add insulation q will increase, if r_0 is greater than k/h, add insulation and q will decrease.

So, this is the concept of critical insulation thickness. So, there exists thickness when if you add insulation the heat flow will increase, go beyond that and you add insulation and the expected things will take place, that is the heat flow will decrease then why it should happen this can only happen when the thickness is very small.

So, when the thickness is very small by adding another layer of insulation you are increasing the resistance for flow of heat through that added layer of insulation; that means, you are increasing the conduction resistance by making the layer thicker; however, with putting the layer of insulation on the outside you are making more area available for conduction because the area available for conduction is simply $2\pi r$ multiplied by length, length is a constant. So, as r increases your area available for convection increases. So, with increase in r the conduction resistance will increase ok.

So, the convective heat transfer is helped by adding insulation because of the additional area conduction heat transfer is going to be reduced by adding the insulation. So, these are 2 parallel mechanisms which compete with each other and for certain value of r as we have been previously equal to k/h the result is you get the maximum heat transfer.

So, if you are below this value of k/h, increase the insulation thickness, increase the area available for convection and that more than offsets the additional conduction resistance that you put in the system that is what the concept of critical insulation thickness is. So, let us just workout with those since r is equal to k/h, what are the typical values of k/h and that would tell us something about when do you expect this concept of critical insulation thickness to be when you are going to encounter such concepts.

So, let us write what are the typical values of k and the typical values of h and then we will know what this is. So, k for insulation material is roughly about 0.0 3 watt per meter Kelvin and the h the convection over here is mostly going to be the convection in air which is the order of 10 watt per metre square per Kelvin. So, r_c the critical thickness of insulation is going to be about 3 millimeters.

So, your size has to be less than 3 millimeter for the concept for the occurrence of the phenomena associated with critical insulation thickness, most of the normal conditions you do not deal with 3 millimeter thin wire or 3 millimeter thin material. So, for most cases your r is going to be more than r_c and therefore, the concept of critical thickness of insulation is not relevant.

So, concept of critical thickness you do not encounter the phenomenon associated with critical insulation thickness since the length scale involved is only about 3 millimeters. So, what we say is that I increase insulation and I reduce heat transfer, but one example I can give you which would show the role critical thickness insulation thickness may play for system.

So, when you think of very thin wires which conduct electricity, the diameter can be less than 3 millimeters. So, when current passes the heat is generated the ohmic heat is the joule, joule heat due to joule heating some amount of heat is generated and you want to dissipate that ok, but at the same time we do not want to live wire without any electrical insulation put on it.

So, what you do is on the thin wire you put a layer of insulation, but what you get is something very interesting you not only make it safe for the wire to be safe since you have put an electrical insulation this electrical insulation is going to act as a thermal insulation as well, because the heat that is generated is going to dissipate to the atmosphere by means of conduction through the electrical insulation and convection to the outside.

Now, if your radius of the electricity carrying wire is less than 3 millimeter, then you are enhancing the heat loss from the system by putting an electrical insulation on top of it. So, you serve 2 purposes, you cover the electrical wire with an insulation and by doing, so, you are increasing the heat transfer and therefore, the wire itself now can be at a lower temperature which will probably prolong its life and will be less hazardous.

So, critical thickness of insulation are mostly relevant in very thin electrical wires where you put an insulation on top of it, but for most of the practical systems since the dimension that we deal with are and the dimensions are more than 3 millimeters you do not get that in most of the applications, but concept wise it is very interesting that it is difficult to it is counter intuitive when I say that by adding insulation I increase the flow of heat.

So, that is something which in this class we have seen that it is a peculiar nature of the phenomena which is a direct result of change in the heat transfer area with radius. As you go outward the area increases and therefore the heat rate remains the same, but heat flux will change.

So, this class we have seen what is radial system, what are the resistances associated with radial systems, what is the peculiar phenomena of critical thickness of insulation. Then in next class we will very quickly go through what is going to be the equivalent form for spherical systems, and something which is known as the overall heat transfer coefficient. And then we will go into systems in which there is going to be generation of heat at steady state and what would be the form of temperature distribution for systems which are generating heat on their own.

So, that would be the topic of the next class.

Heat Transfer
Prof. Sunando Dasgupta
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

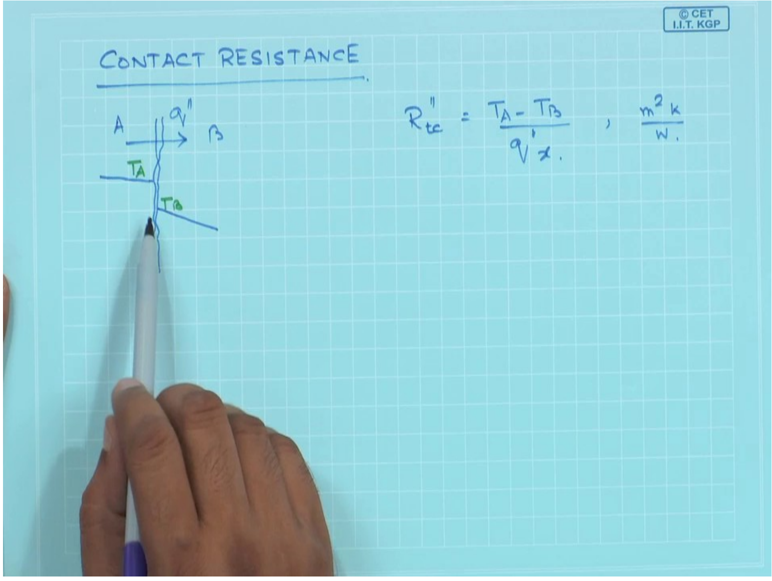
Lecture - 07
Tutorial Problem on Critical Insulation Thickness

We have already been introduced to the concept of critical insulation thickness. What we have seen is an interesting result, which shows that when you add an insulation to a conductor which is generating heat. Normally, we would expect that since we had insulation, the heat flux would decrease, but in some special cases we looked at the equation of heat transfer between a current carrying conductor that is generating heat, with an insulation on top of it. And, we have seen that the heat flow from the conductor to the ambient can be maximized for a specific thickness of the insulation, which is called the critical insulation thickness. We understood that this anomaly can be explained by the simultaneous increase in the heat transfer resistance when you apply the heat, as well as the increase in the surface area available for convection such that the convection resistance would decrease. So, this increase in the resistance to conduction through the insulator, and the decrease in resistance by the availability of enlarged surface area for convective heat transfer, these two we will provide a situation wherein the heat loss from the system would be a maximum. And the thickness where it takes place is known as a critical thickness of insulation. We have also seen that in most of the practical applications, the concept of critical thickness of insulation does not appear. Since the magnitude of critical thickness insulation thickness is less than the order of millimeters. So, therefore, in practical systems we do not come across the critical thickness of insulation, but in some special cases for example, in the case of a current carrying conductor this could be of relevance.

We are going to solve problem in the in this class that would clarify our concept more and so, it is going to be tutorial class based on the concept of critical thickness of insulation, but before we move on to that there is another type of resistance to heat transfer that we have not discussed so, far.

When a composite system consisting of two different materials are brought together, you are never going to get a perfect contact in between the two surfaces, due to the imperfections present i.e. the roughness; on the surfaces> Hene the two surfaces coming onto contact with each other will never have a 100 percent point to point contact at every location. There would exist an area of very low thickness which would be filled by air; and, this air is going to give rise to a significant increase in the heat transfer resistance. So, whenever you bring two surfaces together, due to the roughness present in them, the actual heat transfer resistance is going to be much more than the individual conduction resistances of the two solid blocks. So, this enhanced heat transfer resistance is called sometimes called contact resistance.

(Refer Slide Time: 04:27)

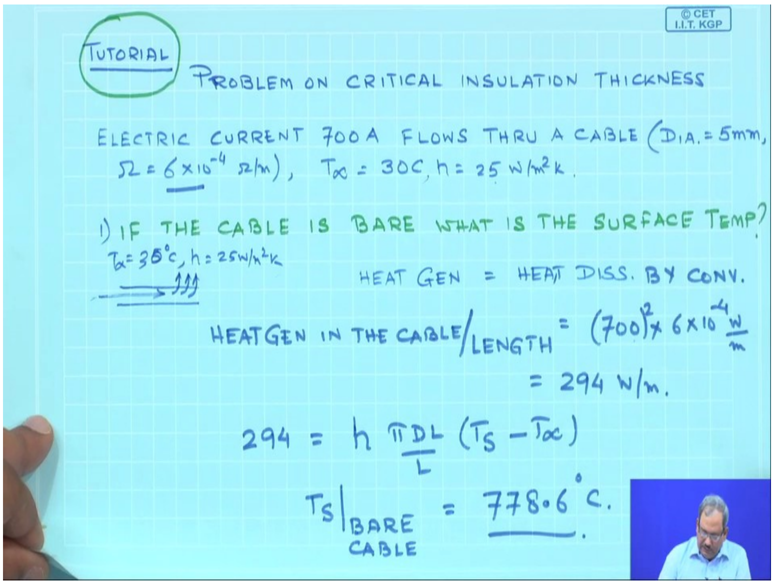


If we look at the figure over here what you see is that the two materials A and B and an at a molecular level they are; obviously, not very not going to be smooth. So, therefore, there would be some amount of air, which is going to be entrapped in the spaces in between these two solid surfaces.

And therefore, if you look at the temperature profile and if T A is the temperature on the A side of the interface and T B is going to be the temperature on the B side of the interface. Due to the presence of the low thermal conductivity air in between these two materials A and B, there is going to be temperature jump across this interface. This temperature jump is expressed in terms of a contact resistance, which as per our previous understanding any resistance is cause, divided by effect. So, the cause is a temperature difference T A minus T B, and the same heat travels through this. So, it is going to T A minus T B by heat flux.

So, when you express the contact resistance in this form; obviously, its units are going to be Kelvin per Watt. In practical systems most of the times you like to reduce the contact resistance, some examples are when two surfaces are melted together. So, as to have very good conductive flow of heat you do not want a thin layer of air existing in between the two. On the other hand, if you are using material B as in that figure as an insulator then having an air pocket in between A and B can be beneficial as it provides an additional resistance. Therefore, depending on your final application the phenomenon of contact resistance can be helpful, or you may not want to have a contact resistance. So, the obvious question therefore, is what is going to be the value of contact resistance? The contact resistance values are significantly more in many cases than the conduction resistances of the two materials, which are brought together. So, one has to take into account the contact resistance in most of the real situations. Now, let us say for some reason you would like to reduce contact resistance. The obvious one is that, you would like to make the two surfaces that you would like to combine as smooth as possible. So, one way of reducing conduct resistance would be to make the surface very smooth. The other option to reduce contact resistance, one may also apply pressure so, as to increase the area of contact in between the two surfaces. So, the two options to reduce contact resistances the resistances are to make the surface smooth and to apply pressure at the contact level. The third option which is also used sometimes is to make sure, that you do not have air in between the two surfaces which are brought into contact, you rather have an interstitial fluid that has a higher thermal conductivity than that of air. You may like to replace the air by a heavy oil with a high thermal conductivity. So, what you do is you coat the two surfaces initially with a heavy oil with high thermal conductivity and then bring the two surfaces together.

(Refer Slide Time: 10:28)



TUTORIAL

PROBLEM ON CRITICAL INSULATION THICKNESS

ELECTRIC CURRENT 700 A FLOWS THRU A CABLE (DIA. = 5mm, $\Omega = 6 \times 10^{-4}\ \Omega/m$), $T_\infty = 30C$, $h = 25\ W/m^2K$.

1) IF THE CABLE IS BARE WHAT IS THE SURFACE TEMP?

$T_\infty = 30^\circ C$, $h = 25 W/m^2K$

HEAT GEN = HEAT DISS. BY CONV.

$$\text{HEAT GEN IN THE CABLE/LENGTH} = (700)^2 \times 6 \times 10^{-4}\ \frac{W}{m}$$

$$= 294\ W/m.$$

$$294 = h \frac{\pi D L}{L} (T_S - T_\infty)$$

$$T_S\Big|_{\substack{\text{BARE}\\ \text{CABLE}}} = \underline{778.6^\circ C}.$$

We now move to the problem that we are going to solve in this class on the concept of critical thickness of insulation. So, the first problem that we are going to do in this, tutorial problem it is about an electric wire through which 700 Ampere of current flows. The diameter of the wire is given as a 5 millimeter, the resistance per unit length is $6 \times 10^{-4}\ \Omega / m$. The electric wire is in an atmosphere where the temperature is 30 ^oC, and the convective heat transfer coefficient from the outside of the wire to the to the atmosphere is 25 to Watt per meter square per Kelvin.

The first part of the problem is if the cable is bare what is going to be its surface temperature? So, you just have this cable where this temperature T infinity is 30 degree centigrade and h is 25 Watt per meter square per Kelvin and you have a current which is flowing through this.

So, at steady state some amount of current is going to be produced and this amount of heat must be dissipated to the atmosphere by the convection process. So, in this case your governing equation heat generated must be equal to the heat that is dissipated by a convection process.

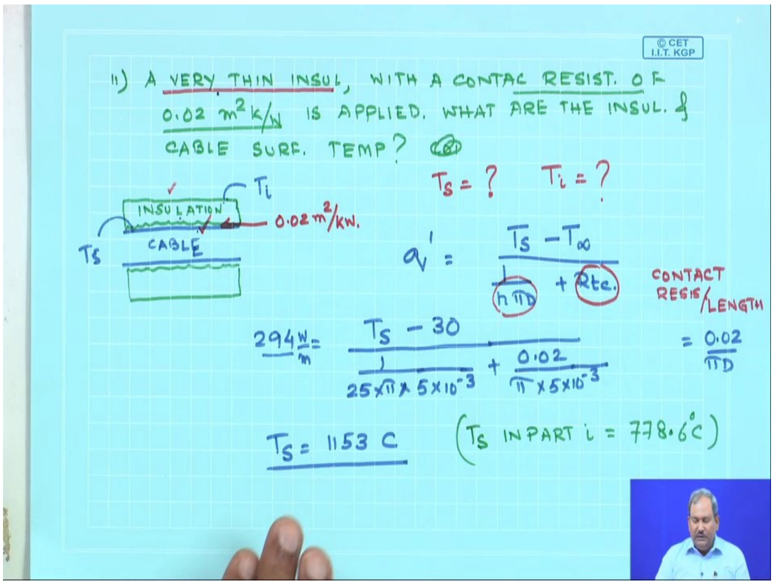
So, what is the heat generation in the cable? So, heat generation in the cable per unit length of the cable is simply going to be I square R. So, it is 700 square times R where R is the R is the resistance per unit length, which is provided in the problem as

$6 \times 10^{-4}\ \Omega / m$. So, the unit of this should be Watt per meter. So, it is the heart generation per unit length in the wire because of this much of current flowing through it.

And you would see that this is going to be 294 Watt per meter heat is generated in here, this generated heat 294 must be equal to the h A by L where A is the area. So, it is pi D L by L. So, this is the heat to which is lost by convection per unit length T of the surface of the wire minus T infinity. So, your value of h is known, which is 25 T infinity known so, you would be able to obtain T S of the bare cable surface to be equal to 778.6 degree centigrade.

So, that is the answer to the first part which simply gives you what is going to be the cable surface temperature, when there is nothing on it.

(Refer Slide Time: 14:02)



ii) A VERY THIN INSUL, WITH A CONTAC RESIST. OF 0.02 m²K/W IS APPLIED. WHAT ARE THE INSUL. & CABLE SURF. TEMP?

$T_S = ?$ $T_i = ?$

$$q' = \frac{T_S - T_\infty}{\frac{1}{h \pi D} + R_{tc}}$$

CONTACT RESIS/LENGTH $= \frac{0.02}{\pi D}$

$$294 \frac{W}{m} = \frac{T_S - 30}{\frac{1}{25 \times \pi \times 5 \times 10^{-3}} + \frac{0.02}{\pi \times 5 \times 10^{-3}}}$$

$T_S = 1153$ C

(T_S IN PART i $= 778.6^\circ C$)

The part 2 of the problem says that a very thin insulation with a contact resistance of 0.02-meter square Kelvin per Watt is applied on the cable, what are the insulation and cable surface temperature?

So, this I have the cable and then on top of the cable I have an insulation, this is a cylindrical system I have this insulation. So, this one is my cable and here I have the insulation. In between the insulation and the cable, and I have some contact resistance. So, what is required is what is the temperature of the cable surface T S, that we have obtained in our previous part of the problem is 778.6, and what is the insulation temperature, what are the insulation and the cable surface temperature, what is the insulation temperature; that means, what is going to be T i? So, these two we have to evaluate now the same amount of current is flowing through the wire but, you have an insulation on top of it right now.

And, the contact resistance is mentioned here. So, we need to consider the contact resistance of 0.02-meter square Kelvin per Watt. So, the question is what is going to be the value of T S and what is going to be the value of T i.

If you think of the situation as if two reservoirs are connected by a pipe. So, if you maintain the level fixed and if you add a pipe in between them there would be definite flow from this reservoir to this reservoir. Now, what you would like to do is you are going to construct the pipe in between. So, you add more resistance to flow in between the two reservoirs. So, the resistance is going to be more, but you would require the same flow rate. So, with the resistance being higher, you would still like the same flow rate then the only option available to you is to increase the height difference between the two reservoirs. So, the potential must be increased if you would like to maintain the same flow in between these two reservoirs with additional resistances in between them.

The same concept would be applicable here as well; the heat generated is the same. So, at steady state the same heat has to go through the insulation through the contact resistance first then through the insulation, and then the heat will be convected out to the atmosphere, but since you have additional resistance the temperature on the wire surface must be increased in order to have the same flow rate in between the two.

I am going to write the equation for the heat flow as the cause divided by the resistance. So, the heat flow I square R remains the same the delta T is unknown to me T of the cable surface minus T infinity. T infinity is known to me, but T of the cable surface is not known to me divided by the sum of resistances. And, what are the resistances; we need to see what resistances we need to consider finding what is the cable surface temperature.

So, let us write the governing equation in here; where this q prime which would be the same as in the previous problem divided by T of S minus T of infinity divided by 1 by h pi d plus Rtc. So, this Rtc is the contact resistance. And, since everything is expressed in terms of per unit length.

The key point is to note in the problem there is a very thin insulation. Since, the insulation thickness is very small I am neglecting the conduction resistance provided by the insulation. So, that is why I do not have the conduction resistance of the insulation included in here. What I only have is the convection resistance at this point and the contact resistance at this point. So, q double prime would simply be equal to a 294 as it was before this T S is unknown to me, I do not know what is the cable surface temperature now, but we realize based on a previous discussion that T S has to be much more than 778, which we have obtained, for the first part of the problem T infinity is 30 and 1 by h is 25 pi times D the diameter of the cable is 5 into 10 to the power minus 3, and the contact resistance point 0.02 is to be divided by pi times D. When you work out this T S is going to be 1153 degree Centigrade; So, compared that with the value of T S, which we have obtained in part 1 which was 778.6 degree Centigrade.

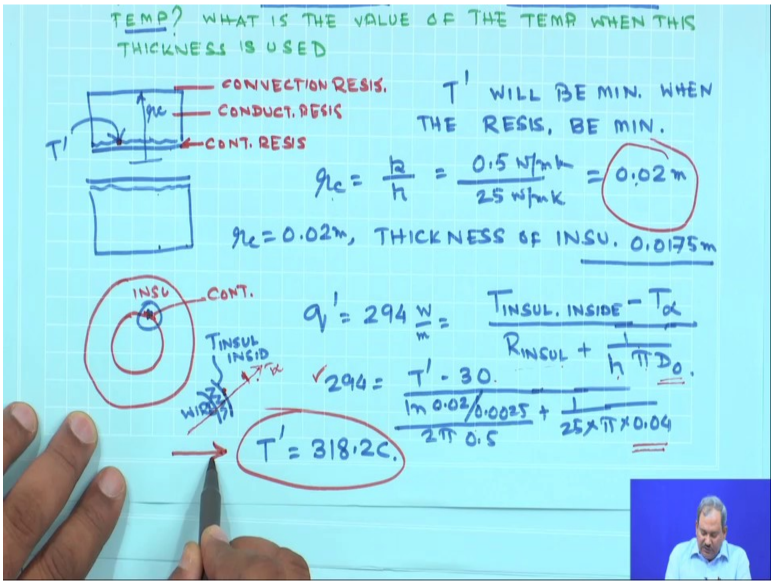
So, as I as I said before, the potential must be increased to have the same flow rate. So, the temperature of the cable surface will now become 1153 to have the same amount of heat flow. So, what is going to be the temperature of the insulation outer surface, remember the insulation is very thin.

Since the insulation is very thin you simply can assume that it has the same diameter as that of the wire. Now, if that is the case you are still going to dissipate 294 Watt per meter of heat from the insulation. So, 294 Watt per meter must be equal to h times pi D, where D is the diameter of the insulation, times T of the insulation minus T infinity.

This D, which is the insulation diameter, since it is very thin it is going it can be taken to be equal to that of the wire. And once you do that your temperature of the outside of the insulation surface will remain identical to it is value of part 1 that is 778.6.

Let us go to the third part of the problem, which is interesting.

(Refer Slide Time: 25:55)



TEMP? WHAT IS THE VALUE OF THE TEMP WHEN THIS THICKNESS IS USED

T' WILL BE MIN. WHEN THE RESIS. BE MIN.

$$r_c = \frac{k}{h} = \frac{0.5\ \text{W/mK}}{25\ \text{W/m}^2\text{K}} = \boxed{0.02\ \text{m}}$$

$r_c = 0.02\text{m}$, THICKNESS OF INSU. 0.0175m

$$q' = 294\ \frac{W}{m} = \frac{T_{\text{INSUL. INSIDE}} - T_\alpha}{R_{\text{INSUL}} + \frac{1}{h \pi D_o}}$$

$$✓\ 294 = \frac{T' - 30}{\frac{\ln 0.02/0.0025}{2\pi\, 0.5} + \frac{1}{25 \times \pi \times 0.04}}$$

$$\boxed{T' = 318.2C}$$

Now, I am going to have a finite thickness of insulation. So, what thickness of insulation, and now I am specifying the value of the k thermal conductivity as 0.5 Watt per meter Kelvin will yield the lowest value of the maximum insulation temperature.

So, what I have now is I have the wire I have the contact resistance in and then I have the insulation, this insulation thickness is now finite. So, here I have contact resistance, here I have conduction resistance and here I have convection resistance.

Now, if you look at this figure it is obvious that the maximum insulation temperature no matter whatever you do is going to be at this point, at the surface where it is connected with the wire in presence of a contact resistance. So, we are trying to find out what is the going to be the maximum value of this contact resistance. Let us call it as T prime maximum value of this temperature. Now, what is going to the minimum value what is going to be the lowest value of this maximum temperature?

Now, the maximum temperature again I use the concept of 2 reservoirs and somehow instead of increasing the resistance of the pipe in between the 2 reservoirs and decreasing the resistance in between the two. So, as I decrease the resistance in between the two in order to have the same flow rate; now I can reduce the height difference to be provided. If I increase the resistance I have to provide more as I have explained before, and if by some means I can reduce the resistance of the connecting pipe then I can bring this down. So, that a less potential difference in this case a less temperature difference has to be provided.

So, how do you reduce the resistance of heat flow when you have an insulation on a wire; obviously, the least resistance possible is going to be the at the situation where you have the critical thickness of insulation. Because based on our previous discussion, we know that when the critic insulation thickness reaches the critical value the resistance will become minimum, and when the resistance is minimum then I can work with a lower potential difference or a lower temperature.

So, coming back to the figure over here, when the thickness of the insulation is equal to the critical insulation thickness, then this T prime is going to have a minimum value. So, T prime will be minimum when the resistance to heat transfer is going to be minimum and the resistance to heat transfer is going to be minimum when your Rc critic insulation thickness is equal to the critical insulation thickness, which by definition is going to be given by k by h, which is 0.5 Watt per meter Kelvin divided by 25 Watt per meter Kelvin is equal to 0.0 2 meter.

So, what this tells is if I can provide the insulation thickness to be equal to 0.02 meter if Rc is less than 0.02 meter then by increasing Rc I decrease the resistance and enhance the flow of heat. So, when my flow is kept constant and if I keep on increasing Rc till I reach the point of 0.02 meter, my resistance will keep on decreasing. And as resistance will keep on decreasing my T prime will keep on decreasing.

So, if I say the T prime which I know is the maximum insulation temperature, this maximum insulation temperature will have its lowest value, when Rc is equal to 0.02 meter.

So, if Rc is 0.02 meter, then the thickness of the insulation of insulation would be 0.0175 meter. So, when you provide a thickness of insulation of 0.01 75 meter; your temperature at the inner surface of the insulation is going to be the least of all other situations and let us find out what is it what it is going to be? So, what I have then is the wire, and then I have the insulation around it and with a contact resistance in here ok. So, if I the heat produced per unit length would still be 294, that is the say Watt per meter that the same heat which is going through it and I am going to write, it between T insulation inside.

So, my 294 would simply be equal to T prime, that is the T insulation inside minus T infinity the T infinity is still 30 and the R insulation is going to be 0.02 by 0.0025 divided by 2 pi 0.5 plus 1 by h is 25 pi times 0.04. So, this would be your this should be the equation and T prime you are going to get as 318.2 degree Centigrade. So, this is very interesting.

I will quickly go through what I have done here, I have written the equation the heat transfer equation between this point, the insulation, and the atmosphere outside. So, between this point and T infinity there are 2 resistances - one is the conduction resistance and the second is the convection resistance. Since I am writing it from this point outwards, the critical insulation thickness does not come into my equation.

The same amount of heat which is generated will pass through the wire, through the contact, through the insulation, to the outside. So, this is what the heat is. So, this 294 in all cases will remain the same. So, what is the, what is the potential it is T insulation inside minus T infinity what are the resistances one is the conductive resistance of the insulation and convective resistance at the outside of the insulation.

So, R insulation for a cylindrical system I use the formula and for convective resistance it simply 1 by h A and since I am expressing it in per unit length it simply going to be 1 by h times pi D 0. This D 0 is now the new D 0, which is based on the critical thickness of insulation. So, when you work out the number this T prime is going to be 318.2. So, you can see is that you have achieved a significant reduction in the temperature by having an insulation on the wire. So, that is the beauty of critical thickness of insulation you have added insulation, but what you get because of it is a reduction in temperature.

MODULE 6

CONVECTION

6.1 Objectives of convection analysis:

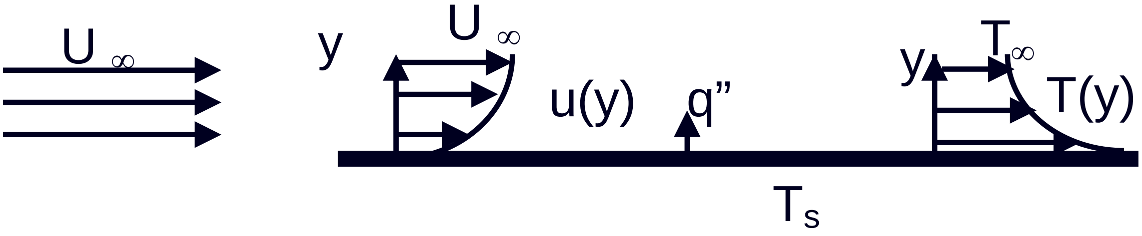
Main purpose of convective heat transfer analysis is to determine:

- flow field
- temperature field in fluid
- heat transfer coefficient, h

How do we determine h ?

Consider the process of convective cooling, as we pass a cool fluid past a heated wall. This process is described by Newton's law of Cooling:

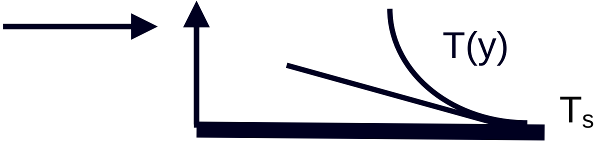
$$q = h \cdot A \cdot (T_S - T_\infty)$$



Near any wall a fluid is subject to the no slip condition; that is, there is a stagnant sub layer. Since there is no fluid motion in this layer, heat transfer is by conduction in this region. Above the sub layer is a region where viscous forces retard fluid motion; in this region some convection may occur, but conduction may well predominate. A careful analysis of this region allows us to use our conductive analysis in analyzing heat transfer. This is the basis of our convective theory.

At the wall, the convective heat transfer rate can be expressed as the heat flux.

$$q''_{conv} = -k_f \left.\frac{\partial T}{\partial y}\right)_{y=0} = h\left(T_s - T_\infty\right)$$

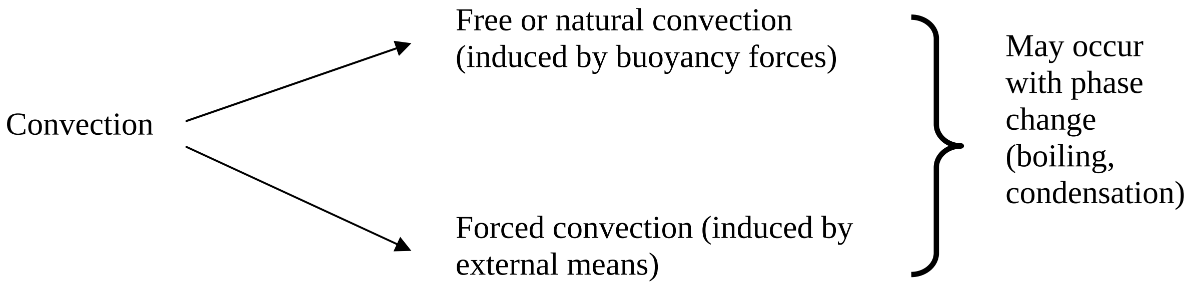


Hence, $h = \dfrac{-k_f \left.\dfrac{\partial T}{\partial y}\right)_{y=0}}{\left(T_s - T_\infty\right)}$

But $\left.\dfrac{\partial T}{\partial y}\right)_{y=0}$ depends on the whole fluid motion, and both fluid flow and heat transfer equations are needed

The expression shows that in order to determine h, we must first determine the temperature distribution in the thin fluid layer that coats the wall.

2.2 Classes of Convective Flows



- extremely diverse
- several parameters involved (fluid properties, geometry, nature of flow, phases etc)
- systematic approach required
- classify flows into certain types, based on certain parameters
- identify parameters governing the flow, and group them into **meaningful non-dimensional numbers**
- need to understand the physics behind each phenomenon

Common classifications:
A. ***Based on geometry:***
External flow / Internal flow
B. ***Based on driving mechanism***
Natural convection / forced convection / mixed convection
C. ***Based on number of phases***
Single phase / multiple phase
D. ***Based on nature of flow***
Laminar / turbulent

Table 6.1. Typical values of h (W/m^2K)

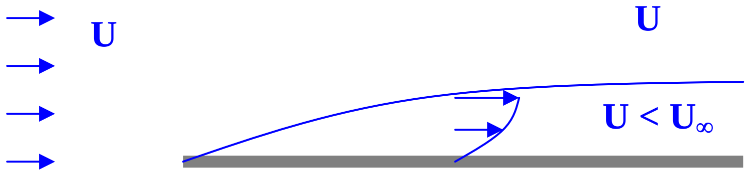
Free convection	gases: 2 - 25 liquid: 50 – 100
Forced convection	gases: 25 - 250 liquid: 50 - 20,000
Boiling/Condensation	2500 -100,000

2.3 How to solve a convection problem ?

- Solve governing equations along with boundary conditions
- Governing equations include
  1. conservation of mass
  2. conservation of momentum
  3. conservation of energy
- In Conduction problems, only (3) is needed to be solved. Hence, only *few parameters* are involved
- In Convection, all the governing equations need to be solved.
  ⇒ large number of parameters can be involved

2.4 FORCED CONVECTION: external flow (over flat plate)

An internal flow is surrounded by solid boundaries that can restrict the development of its boundary layer, for example, a pipe flow. An external flow, on the other hand, are flows over bodies immersed in an unbounded fluid so that the flow boundary layer can grow freely in one direction. Examples include the flows over airfoils, ship hulls, turbine blades, etc



- Fluid particle adjacent to the solid surface is at rest
- These particles act to retard the motion of adjoining layers
- ⇒ boundary layer effect

Inside the boundary layer, we can apply the following conservation principles:
Momentum balance: inertia forces, pressure gradient, viscous forces, body forces
Energy balance: convective flux, diffusive flux, heat generation, energy storage

2.5 Forced Convection Correlations

Since the heat transfer coefficient is a direct function of the temperature gradient next to the wall, the physical variables on which it depends can be expressed as follows:
h=f(fluid properties, velocity field ,geometry,temperature etc.)

As the function is dependent on several parameters, the heat transfer coefficient is usually expressed in terms of **correlations involving pertinent non-dimensional numbers**.

Forced convection: **Non-dimensional groupings**

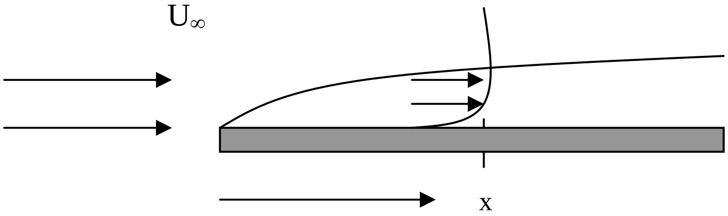
- **Nusselt No**. $Nu = hx / k$ = (convection heat transfer strength)/ (conduction heat transfer strength)
- **Prandtl No.** $Pr = \nu/\alpha$ = (momentum diffusivity)/ (thermal diffusivity)
- **Reynolds No**. $Re = U x / \nu$ = (inertia force)/(viscous force)

Viscous force provides the dampening effect for disturbances in the fluid. If dampening is strong enough ⇒ **laminar flow**
Otherwise, instability ⇒ **turbulent flow ⇒ critical Reynolds number**

For forced convection, the heat transfer correlation can be expressed as

$$Nu=f\ (Re, Pr)$$

The convective correlation for laminar flow across a flat plate heated to a constant wall temperature is:



$$Nu_x = 0.323 \cdot Re_x^{1/2} \cdot Pr^{1/3}$$

where

$$Nu_x \equiv h \cdot x/k$$

$$Re_x \equiv (U_\infty \cdot x \cdot \rho)/\mu$$

$$Pr \equiv c_P \cdot \mu/k$$

Physical Interpretation of Convective Correlation

The Reynolds number is a familiar term to all of us, but we may benefit by considering what the ratio tells us. Recall that the thickness of the dynamic boundary layer, δ, is proportional to the distance along the plate, x.

$$Re_x \equiv (U_\infty \cdot x \cdot \rho)/\mu \ \propto\ (U_\infty \cdot \delta \cdot \rho)/\mu \ =\ (\rho \cdot U_\infty^2)/(\mu \cdot U_\infty/\delta)$$

The numerator is a mass flow per unit area times a velocity; i.e. a momentum flow per unit area. The denominator is a viscous stress, i.e. a viscous force per unit area. The ratio represents the ratio of momentum to viscous forces. If viscous forces dominate, the flow will be laminar; if momentum dominates, the flow will be turbulent.

Physical Meaning of Prandtl Number

The Prandtl number was introduced earlier.

If we multiply and divide the equation by the fluid density, ρ, we obtain:

$$Pr \equiv (\mu/\rho)/(k/\rho \cdot c_P) = \upsilon/\alpha$$

The Prandtl number may be seen to be a ratio reflecting the ratio of the rate that viscous forces penetrate the material to the rate that thermal energy penetrates the material. As a consequence the Prandtl number is proportional to the rate of growth of the two boundary layers:

$$\delta/\delta_t = Pr^{1/3}$$

Physical Meaning of Nusselt Number

The Nusselt number may be physically described as well.

$$Nu_x \equiv h \cdot x/k$$

If we recall that the thickness of the boundary layer at any point along the surface, δ, is also a function of x then

$$Nu_x \propto h \cdot \delta/k \propto (\delta/k \cdot A)/(1/h \cdot A)$$

We see that the Nusselt may be viewed as the ratio of the conduction resistance of a material to the convection resistance of the same material.

Students, recalling the Biot number, may wish to compare the two so that they may distinguish the two.

$$Nu_x \equiv h \cdot x/k_{fluid} \qquad\qquad Bi_x \equiv h \cdot x/k_{solid}$$

The denominator of the Nusselt number involves the thermal conductivity of the **fluid** at the solid-fluid convective interface; The denominator of the Biot number involves the thermal conductivity of the **solid** at the solid-fluid convective interface.

Local Nature of Convective Correlation

Consider again the correlation that we have developed for laminar flow over a flat plate at constant wall temperature

$$Nu_x = 0.323 \cdot Re_x^{1/2} \cdot Pr^{1/3}$$

To put this back into dimensional form, we replace the Nusselt number by its equivalent, hx/k and take the x/k to the other side:

$$h = 0.323 \cdot (k/x) \cdot Re_x^{½} \cdot Pr^{1/3}$$

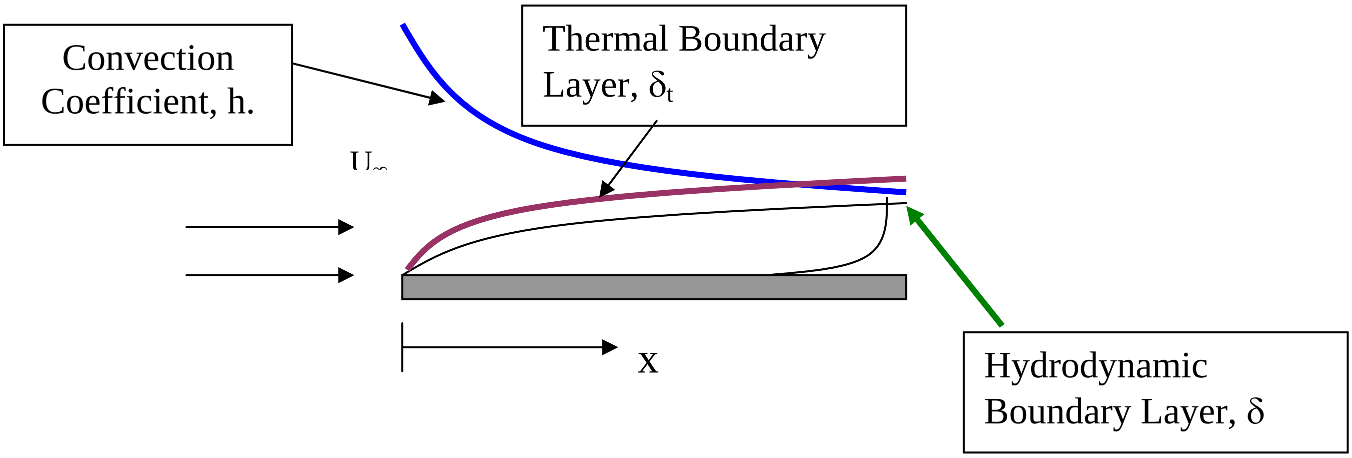
Now expand the Reynolds number

$$h = 0.323 \cdot (k/x) \cdot [(U_\infty \cdot x \cdot \rho)/\mu]^{½} \cdot Pr^{1/3}$$

We proceed to combine the x terms:

$$h = 0.323 \cdot k \cdot [(U_\infty \cdot \rho)/(x \cdot \mu)]^{½} \cdot Pr^{1/3}$$

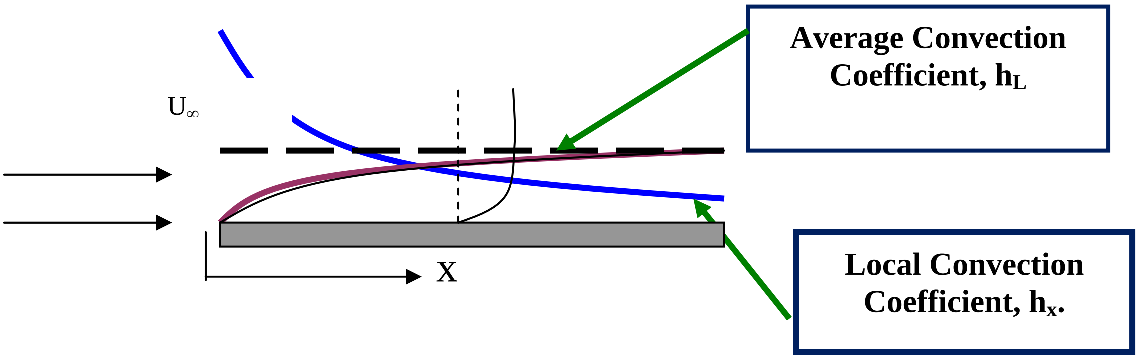
And see that the convective coefficient decreases with $x^{½}$.



We see that as the boundary layer thickens, the convection coefficient decreases. Some designers will introduce a series of “trip wires”, i.e. devices to disrupt the boundary layer, so that the buildup of the insulating layer must begin anew. This will result in regular “thinning” of the boundary layer so that the convection coefficient will remain high.

Averaged Correlations

If one were interested in the total heat loss from a surface, rather than the temperature at a point, then they may well want to know something about average convective coefficients.



The desire is to find a correlation that provides an overall heat transfer rate:

$$Q = h_L \cdot A \cdot [T_{wall} - T_\infty] = \int h_x \cdot [T_{wall} - T_\infty] \cdot dA = \int_0^L h_x \cdot [T_{wall} - T_\infty] \cdot dx$$

where h_x and h_L, refer to local and average convective coefficients, respectively.

Compare the second and fourth equations where the area is assumed to be equal to A = (1·L):

$$h_L \cdot L \cdot [T_{wall} - T_\infty] = \int_0^L h_x \cdot [T_{wall} - T_\infty] \cdot dx$$

Since the temperature difference is constant, it may be taken outside of the integral and cancelled:

$$\boxed{h_L \cdot L = \int_0^L h_x \cdot dx}$$

This is a general definition of an integrated average.

Proceed to substitute the correlation for the local coefficient.

$$h_L \cdot L = \int_0^L 0.323 \cdot \frac{k}{x} \cdot \left[\frac{U_\infty \cdot x \cdot \rho}{\mu}\right]^{0.5} \cdot \mathrm{Pr}^{1/3} \cdot dx$$

Take the constant terms from outside the integral, and divide both sides by k.

$$h_L \cdot L/k \;=\; 0.323 \cdot \left[\frac{U_\infty \cdot \rho}{\mu}\right]^{0.5} \cdot \mathrm{Pr}^{1/3} \cdot \int_0^L \left[\frac{1}{x}\right]^{0.5} \cdot dx$$

Integrate the right side.

$$h_L \cdot L/k = 0.323 \cdot \left[\frac{U_\infty \cdot \rho}{\mu}\right]^{0.5} \cdot \mathrm{Pr}^{1/3} \cdot \left.\frac{x^{0.5}}{0.5}\right|_0^L$$

The left side is defined as the average Nusselt number, Nu_L. Algebraically rearrange the right side.

$$Nu_L = \frac{0.323}{0.5} \cdot \left[\frac{U_\infty \cdot \rho}{\mu}\right]^{0.5} \cdot \mathrm{Pr}^{\frac{1}{3}} \cdot L^{0.5} = 0.646 \cdot \left[\frac{U_\infty \cdot L \cdot \rho}{\mu}\right]^{0.5} \cdot \mathrm{Pr}^{\frac{1}{3}}$$

The term in the brackets may be recognized as the Reynolds number, evaluated at the end of the convective section. Finally,

$$\boxed{\mathrm{Nu_L} = 0.646 \cdot \mathrm{Re}_L^{0.5} \cdot \mathrm{Pr}^{1/3}}$$

This is our average correlation for laminar flow over a flat plate with constant wall temperature.

Reynolds Analogy

In the development of the boundary layer theory, one may notice the strong relationship between the dynamic boundary layer and the thermal boundary layer. Reynold's noted the strong correlation and found that fluid friction and convection coefficient could be related. This is known as the Reynolds Analogy.

Conclusion from Reynold's analogy: Knowing the frictional drag, we know the Nusselt Number. If the drag coefficient is increased, say through increased wall roughness, then the convective coefficient will also increase.

Turbulent Flow

We could develop a turbulent heat transfer correlation in a manner similar to the von Karman analysis. It is probably easier, having developed the Reynolds analogy, to follow that course. The local fluid friction factor, C_f, associated with turbulent flow over a flat plate is given as:

$$C_f = 0.0592/\mathrm{Re}_x^{0.2}$$

Substitute into the Reynolds analogy:

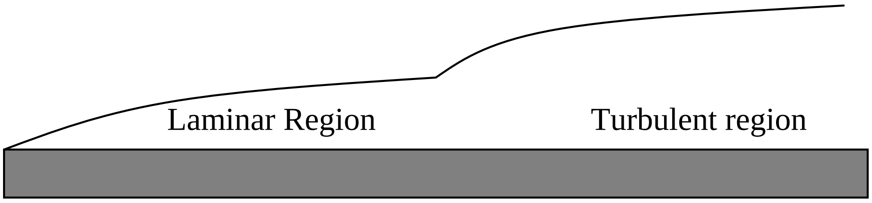
$$(0.0592/\mathrm{Re}_x^{0.2})/2 = \mathrm{Nu}_x/\mathrm{Re}_x\mathrm{Pr}^{1/3}$$

Rearrange to find

$$\mathrm{Nu}_x = 0.0296 \cdot \mathrm{Re}_x^{0.8} \cdot \mathrm{Pr}^{1/3}$$

Local Correlation Turbulent Flow Flat Plate.

In order to develop an average correlation, one would evaluate an integral along the plate similar to that used in a laminar flow:



$$h_L \cdot L = \int_0^L h_x dx = \int_0^{L_{crit}} h_{x,laminar} \cdot dx + \int_{L_{crit}}^{L} h_{x,turbulent} \cdot dx$$

Note: The critical Reynolds number for flow over a flat plate is $5 \cdot 10^5$; the critical Reynolds number for flow through a round tube is 2000.

The result of the above integration is:

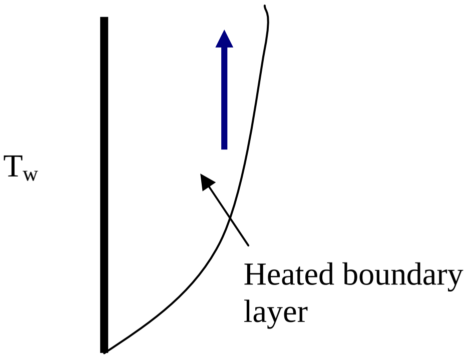
$$Nu_x = 0.037 \cdot (Re_x^{0.8} - 871) \cdot Pr^{1/3}$$

Note: Fluid properties should be evaluated at the average temperature in the boundary layer, i.e. at an average between the wall and free stream temperature.

$$T_{prop} = 0.5 \cdot (T_{wall} + T_{\infty})$$

2.6 Free convection

Free convection is sometimes defined as a convective process in which fluid motion is caused by buoyancy effects.

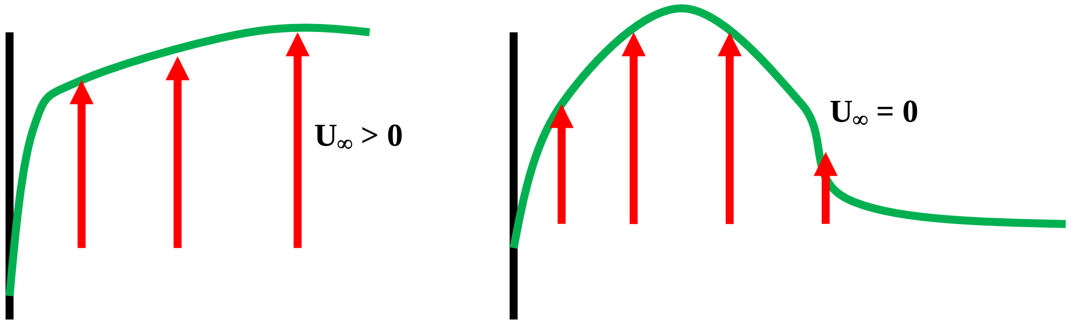


$T_\infty < T_{\text{boundry. layer}} < T_w$

$\rho_\infty < \rho_{\text{boundry. layer}}$

Velocity Profiles

Compare the velocity profiles for forced and natural convection shown below:



Forced Convection **Free Convection**

Coefficient of Volumetric Expansion

The thermodynamic property which describes the change in density leading to buoyancy in the Coefficient of Volumetric Expansion, β.

$$\beta \equiv -\frac{1}{\rho}\cdot\left.\frac{\partial \rho}{\partial T}\right|_{P=Const.}$$

Evaluation of β

- Liquids and Solids: β is a thermodynamic property and should be found from Property Tables. Values of β are found for a number of engineering fluids in Tables given in Handbooks and Text Books.
- Ideal Gases: We may develop a general expression for β for an ideal gas from the ideal gas law:

$$P = \rho \cdot R \cdot T$$

Then,

$$\rho = P/R \cdot T$$

Differentiating while holding P constant:

$$\left.\frac{d\rho}{dT}\right|_{P=Const.} = -\frac{P}{R \cdot T^2} = -\frac{\rho \cdot R \cdot T}{R \cdot T^2} = -\frac{\rho}{T}$$

Substitute into the definition of β

$$\beta = \frac{1}{T_{abs}}$$

Ideal Gas

Grashof Number

Because U_∞ is always zero, the Reynolds number, $[\rho \cdot U_\infty \cdot D]/\mu$, is also zero and is no longer suitable to describe the flow in the system. Instead, we introduce a new parameter for natural convection, the Grashof Number. Here we will be most concerned with flow across a vertical surface, so that we use the vertical distance, z or L, as the characteristic length.

$$\boxed{Gr \equiv \frac{g \cdot \beta \cdot \Delta T \cdot L^3}{\nu^2}}$$

Just as we have looked at the Reynolds number for a physical meaning, we may consider the Grashof number:

$$Gr \equiv \frac{\rho^2 \cdot g \cdot \beta \cdot \Delta T \cdot L^3}{\mu^2} = \frac{\left(\frac{\rho \cdot g \cdot \beta \cdot \Delta T \cdot L^3}{L^2}\right) \cdot (\rho \cdot U_{\max}^2)}{\mu^2 \cdot \frac{U_{\max}^2}{L^2}} = \frac{\left(\frac{Buoyant\,Force}{Area}\right) \cdot \left(\frac{Momentum}{Area}\right)}{\left(\frac{Viscous\,Force}{Area}\right)^2}$$

Free Convection Heat Transfer Correlations

The standard form for free, or natural, convection correlations will appear much like those for forced convection except that (1) the Reynolds number is replaced with a Grashof number and (2) the exponent on Prandtl number is not generally 1/3 (The von Karman boundary layer analysis from which we developed the 1/3 exponent was for forced convection flows):

$Nu_x = C \cdot Gr_x^m \cdot Pr^n$ Local Correlation

$Nu_L = C \cdot Gr_L^m \cdot Pr^n$ Average Correlation

Quite often experimentalists find that the exponent on the Grashof and Prandtl numbers are equal so that the general correlations may be written in the form:

$Nu_x = C \cdot [Gr_x \cdot Pr]^m$ Local Correlation

$Nu_L = C \cdot [Gr_L \cdot Pr]^m$ Average Correlation

This leads to the introduction of the new, dimensionless parameter, the Rayleigh number, Ra:

$Ra_x = Gr_x \cdot Pr$

$$Ra_L = Gr_L \cdot Pr$$

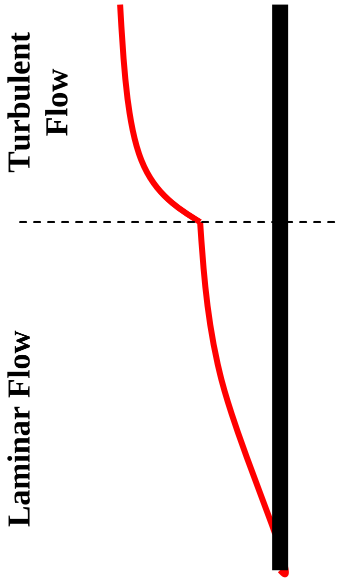
So that the general correlation for free convection becomes:

$\boxed{Nu_x = C \cdot Ra_x^m}$ Local Correlation

$\boxed{Nu_L = C \cdot Ra_L^m}$ Average Correlation

Laminar to Turbulent Transition

Just as for forced convection, a boundary layer will form for free convection. The boundary layer, which acts as a thermal resistance, will be relatively thin toward the leading edge of the surface resulting in a relatively high convection coefficient. At a Rayleigh number of about 10^9 the flow over a flat plate will become transitional and finally become turbulent. The increased turbulence inside the boundary layer will enhance heat transfer leading to relative high convection coefficients because of better mixing.



$Ra < 10^9$ Laminar flow. [Vertical Flat Plate]

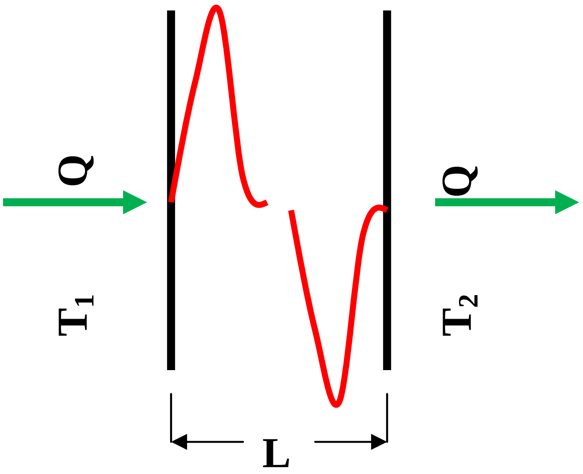
$Ra > 10^9$ Turbulent flow. [Vertical Flat Plate]

Generally the characteristic length used in the correlation relates to the distance over which the boundary layer is allowed to grow. In the case of a vertical flat plate this will be x or L, in the case of a vertical cylinder this will also be x or L; in the case of a horizontal cylinder, the length will be d.

Critical Rayleigh Number

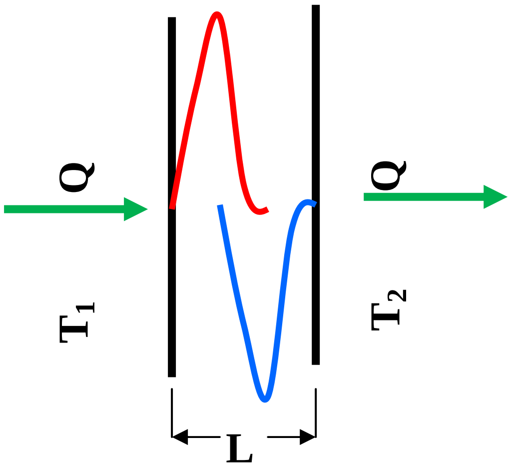
Consider the flow between two surfaces, each at different temperatures. Under developed flow conditions, the interstitial fluid will reach a temperature between the temperatures of the two surfaces and will develop free convection flow patterns. The fluid will be heated by one surface, resulting in an upward buoyant flow, and will be cooled by the other, resulting in a downward flow.

Note that for enclosures it is customary to develop correlations which describe the overall (both heated and cooled surfaces) within a single correlation.

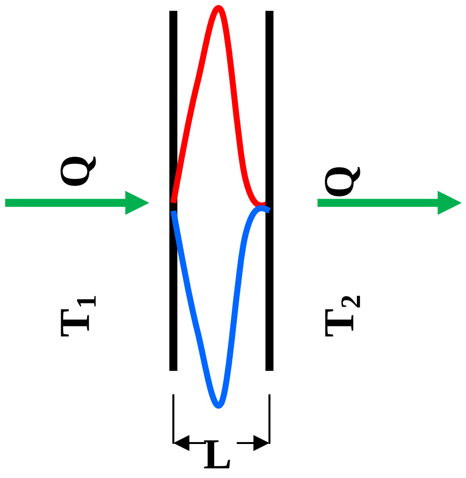


Free Convection Inside an Enclosure (boundary layer limit)

If the surfaces are placed closer together, the flow patterns will begin to interfere:



Free Convection Inside an Enclosure With Partial Flow Interference



Free Convection Inside an Enclosure With Complete Flow Interference (Channel flow limit)

In the case of complete flow interference, the upward and downward forces will cancel, canceling circulation forces. This case would be treated as a pure convection problem since no bulk transport occurs.

The transition in enclosures from convection heat transfer to conduction heat transfer occurs at what is termed the "**Critical Rayleigh Number**". Note that this terminology is in clear contrast to forced convection where the critical Reynolds number refers to the transition from laminar to turbulent flow.

$Ra_{crit} = 1000$ (Enclosures With Horizontal Heat Flow)

$Ra_{crit} = 1728$ (Enclosures With Vertical Heat Flow)

The existence of a Critical Rayleigh number suggests that there are now three flow regimes: (1) No flow, (2) Laminar Flow and (3) Turbulent Flow. In all enclosure problems the Rayleigh number will be calculated to determine the proper flow regime before a correlation is chosen.

Heat Transfer
Prof. Sunando Dasgupta
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 16
Fundamentals of Convection

We are going to start with another mode of Heat Transfer namely convection, which is almost omnipresent whenever there is a contact between a solid and a fluid and the solid temperature and fluid temperature are different. Now, in addition we may have the fluid, have a velocity with which it flows over the solid. So, in that case the heat transfer, the major heat transfer, the maximum amount of heat transfer is going to take place through convection. So, convection requires the presence of a medium and in most of the cases there would be an imposed flow of the fluid over the solid.

So, convection is characterized by the presence of a moving fluid stream, in contact with a solid. In some special cases, there maybe this motion of the fluid can be induced by a density difference by a, by a buoyant force without the presence of an external agency that drives the fluid over the solid surface, which is known as natural convection or free convection. So, an object which is or a hot object which is placed in, let us say water, it is going to lose its heat by convection and unless there is a flow imposed flow of the water stream. Then it is going to be natural or free convection, where the liquid near the solid, its temperature will increase, its density will decrease and due to the buoyant force that hot liquid will rise up along the solid and to be replaced by cold water from the surrounding.

So, a natural current would therefore, set in which is known as natural convection. So, we will treat natural convection separately towards the end of our discussion on convective heat transfer. But right now we are going to mostly concentrate on situations in which there is an imposed motion of the fluid in contact with the solid, the applications of convective heat transfer or the occurrence of convective heat transfer is everywhere in industrial processes. So, wherever you have to heat up a fluid which is entering a reactor or you would like to cool a stream of liquid before you discharge it to somewhere, it requires the exchange of heat in between two fluid streams. It is also very important to ensure that the chemical plant is going to operate at its highest efficiency, if you can regenerate some of the heat which would otherwise be lost.

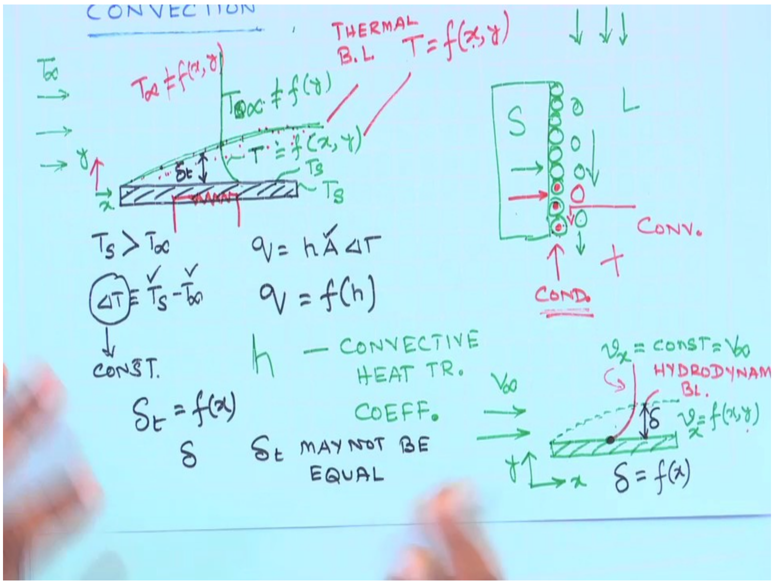
So, convection plays an important role in how you can effectively design an equipment, which is going to have the maximum amount of convective heat transfer between a solid and the fluid or between two fluids streams separated by a solid barrier. These considerations play an integral role in evaluating, in determining the efficiency of the process. So, the heat exchangers that we would again consider towards, on in this course the heat exchangers, the design operating principle of heat exchangers they rely heavily on convective heat transfer.

So, it is important that we understand and learn convection, but in order to have a formal mechanism to study convective heat transfer one has to start with the simplest possible case. Where, we have a solid plate which is at a higher temperature in relative to the fluid which, with which it is in contact. And as I said we are going to concentrate mostly on forced convective heat transfer; that means, the fluid which is in contact with the fluid with the solid is moving with a certain velocity.

Now whenever a fluid comes in contact with a solid which is stationary, then there is going to be a change in the hydrodynamic pattern of the fluid which is flowing over it. So, we will discuss that and the concept of boundary layers would be relevant in describing the convective heat transfer, that is expected out of the solid plate, out of the flat solid plate. So, we have chosen flat solid plate because it is the simplest possible geometry that you can think of. Any change any curvature in the solid with which the liquid is in contact will give rise to additional complexity, which is slightly more complex. So, our starting point would be, we would like to see how the heat transfer from a solid plate would take place when it is in contact with water. Let us say the fluid is water, which is very common which we see almost every day.

So, what is going to happen to it? So, we start with the fundamental concepts of boundary layer, some of you I think most of you already are aware of the concept of boundary layer, when we considered the momentum transfer part of it.

(Refer Slide Time: 05:49)



So, let us say this is a solid, which we have, I have said it is a temperature of T_s and this stream of fluid is approaching the solid with a temperature of T_∞ and let us assume that T_s is greater than T_∞. So, the fluid is going to come in contact with the solid, extract heat by the convection process from the solid and as a result of which the temperature of the solid, temperature of the liquid in the vicinity of the solid will rise.

So, this increase in the temperature of the fluid very close to the solid plate is something that we need to model in order to get an idea of what is the heat transfer coefficient. It was the principle equation, the relation that we are going to follow, that we would we are going to use extensively in describing convective heat transfer is Newton's law of cooling, which simply tells us the amount of heat, which is lost from the solid in this case, is can be written as $q = hA\Delta T$, where h is the convective heat transfer coefficient, A is the area in contact and ΔT is the temperature difference between the solid and that of the liquid, which is at a point far from the solid.

So, when I look at this one, this the amount of heat lost from the solid would simply be equal to $hA\Delta T$, where ΔT is defined as $T_s - T_\infty$ and if there is a mechanism, if, let us say there is a heater which maintains, which maintains the temperature of this, of the solid at constant at T_s then T_s is a constant the temperature with which the liquid is coming in contact or approaching the solid that is also a constant. So, ΔT is simply going to be a constant and if ΔT is a constant the area which is in contact with the fluid that is also a constant. So, essentially the heat q that is there is lost from the solid is going to be a function of h.

So, how can we manipulate h, the convective heat transfer coefficient or how can we evaluate the convective heat transfer coefficient that becomes the study of convective heat transfer. So, how do I relate h with other parameters, now what are the parameters on which h would depend on. So, if you think heuristically, let us see the fluid if it is moving at a higher velocity, all of us realize that when you are outside and a cold wind blows on a winter day, you feel more cold as compared to the case where the air velocity has significantly reduced.

So, the temperature in both cases will remain the same; however, the velocity will be able to extract, because of the velocity the cold air would be able to extract more energy from your body and thereby you would feel cooler. So, h, the convective heat transfer coefficient is going to be a strong function of velocity, convective heat transfer coefficient would also depend on the thermo physical properties of the fluid which is flowing over the solid.

So, what are the thermo physical properties on which it would depend on, since we talk about flow, the two properties which come to our mind automatically, one is what is its viscosity, that is an important that will play an important role in what is going to be its density. So, ρ and μ would come into any expression of h, specially when we concentrate only on the momentum transfer part of it; that means, only fluid flow part of it.

Now, this heat is being taken up by the fluid by as sensible heat. So, the temperature of the fluid in contact with the solid will start to rise and whenever you are going to have an increase in temperature of the fluid, the capacity that heat, the thermal capacity of the fluid must be taken into account. And the one of the parameters, one of the properties which define the thermal capacity of the fluid in terms of extracting heat from the solid has to be Cp which is the specific heat.

So, Cp is going to play a major role and how is heat going to transfer between the solid and the liquid. So, the molecules of the fluid which are flowing over the solid, when they come in contact with the solid due to the no slip condition. Which I am sure you are aware of from your fluid mechanics study; that means, the molecules of the fluid, molecules of the moving fluid, which are in contact with the solid they do not move so, they become static. So, there would be a decrease in velocity as we approach the solid and on the solid the velocity of the fluid would be equal to 0, which is the no slip condition.

So, at the interface, at the solid liquid interface there would be solid in one side and static molecules of liquid on the other side. So, if I draw it, if this is my solid part of it, if this is the solid and this is the liquid. Then even if the liquid is flowing some the molecules of the liquid, which are in contact of with the solid due to no slip condition, they have a 0 velocity. Now, when they have a 0 velocity, so the heat is going to get transferred from the solid to this static liquid molecules by conduction, because, if you remember the conduction is prevalent when there is no motion of the molecules in contact.

So, the mechanism by which heat gets transferred from the solid to the liquid molecules, solid to the static liquid molecules is by conduction, but the molecules out of this, molecules beyond this static layer they have a velocity. As they have a velocity now, the heat transfer between two liquid molecules one which is static and one which is moving the heat transfer here is going to be by convection. So, conduction and convection both will exist in order to have convective heat transfer from the solid to the liquid.

So, we understand that an important part here is that you can never have convection without conduction. So, you have to have conduction through the layer, through the static layer of the fluid molecules which are clinging to the side of the solid and thereby having gaining heat by from the solid by conduction. On the other hand, the other side they are exposed to mobile liquid molecules and these, the interaction between the static liquid and the mobile liquid the heat transfer is going to be by convection. So, when we say that a convection from the solid takes place, we need to understand that the convection is going to be preceded by conduction so the layer of the immobile molecules. So, we can never have convection without conduction.

So, let us now concentrate on what is going to happen to the liquid or to the fluid which is coming in contact with the solid. So, the molecules which are over here, there going to get heated and, but the molecule at this point they do not know that a hot plate exists. So, as I move to some more distance, the molecules at a slightly higher depth if this I call as my depth, at a slightly higher depth would know that there is a hot plate which exists. And the extent of the influence of the solid plate would propagate more and more into the liquid as I move in the direction, in the x direction. So, this is my x direction and this is my y direction. So, for small values of x, the effect of the plate in terms of a change in temperature is going to be limited to a point very close to the surface, as I move further with x the depth the penetration of the temperature front, will be more and more into the liquid.

So, if I approximately join them together, I am going to get a layer which more or less demarcates the range where you would expect a change in temperature. So, the temperature over here is simply going to be T_∞, the temperature over here is T_s. So, in between this point and this point there is going to be sharp change in temperature from T_∞ to T_s and if you go beyond this, beyond this point the temperature everywhere is going to be equal to T_∞. So, the temperature profile probably would look something like this, vertical because T_∞ is not a function of y for the region beyond this imaginary layer and here T the temperature is going to be a function of both x and y. So, inside this layer, I would write it clearly again T is a function both of x and y.

So, further you are in terms of x, the temperature is going to be more further you go away from the solid, the temperature will reduce. So, therefore, t is a function both of x and y, but in here T_∞ is not a function of either x or y. So, this line imaginary, line which demarcates the change in temperature field and a constant temperature field due to the effect of convective heat transfer from a solid to a flowing fluid is known as the boundary layer, thermal boundary layer. You are aware of what is a hydrodynamic boundary layer, which is defined in the same way so if I, just to recap some of these fluid mechanics part of it which is going to be required for our study of convective heat transfer. If this is my plate, where I have flow, which let us say it is the velocity is V_∞ then there would be an imaginary layer like this there, would be a layer like this in which the velocity this is my x and this is y.

So, in here the velocity is going to be a function of both x and y, out here the velocity this is a x component of velocity V_x is going to be a constant and for a flat plate this should be equal to V_∞. So, for a so this layer which is in here, the velocity varies from 0 due to no slip condition at the solid it increases asymptotically and then becomes a constant. So, this is a V_x over here which is a constant, but in here the velocity is a function of both x and y. So, all the effects of convective or flow is confined within this layer and most of these layers are, if these layers which are called boundary layers, why they called boundary layers because they demarcate between two different types of flow.

When we considered the flow over a flat plate, whose temperature is equal to the temperature of the fluid, no heat transfer is taking place, but the only thing that is happening is that due to no slip condition, on the solid the velocity is 0. And as you move away from the solid the effect of the solid will be felt lesser and lesser as you move away from the solid and after a certain point, the velocity or the moving fluid will not realize that there exists a stagnant solid plate below it.

So, the layer up to which the effect of the solid is filled by the moving fluid is known as the hydrodynamic boundary layer and of course, the motion of the fluid molecules slipping past one another near the solid plate, the property of relevance is; obviously, viscosity. So, the viscosity is the one which transfers the presence, the effect of the presence of the plate into greater depths of the fluid. So, viscosity in ρ they play a very important role in defining what is the hydrodynamic boundary layer thickness, which are going to, which are generally very small of the order of a few millimeters for normal sized plates.

So, within this few millimeters, the effect of viscosity is important, outside of this boundary layer the viscous effects are unimportant and the entire flow can be treated as inviscid flow. A flow in which the viscosity can be assumed to be equal to 0 because there is no momentum transfer in a direction perpendicular to the flow and therefore, it is inviscid in nature. However, inside the thin layer close to the solid surface the effect of viscosity cannot be neglected.

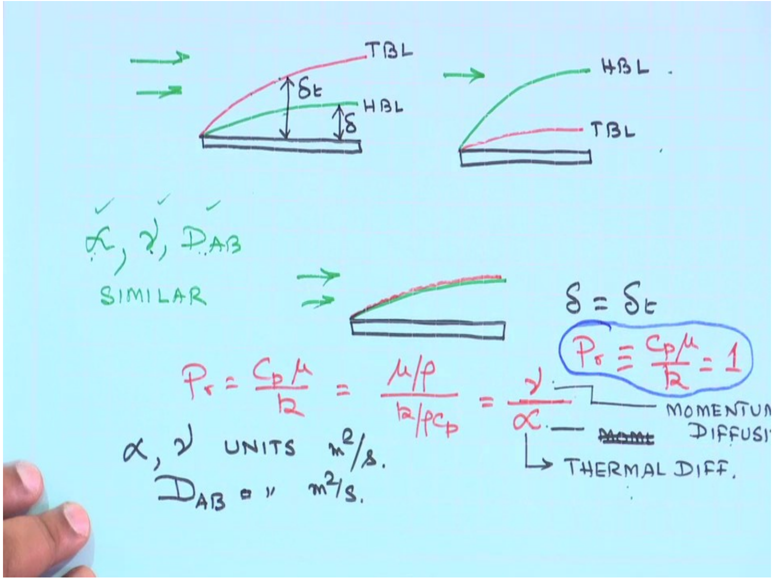
So, we have viscous flow inside the boundary layer, hydrodynamic boundary layer and inviscid flow outside of the boundary layer. Now, from your fluid mechanics you probably also remember that in an inviscid flow, can be explained by Euler’s equation. In Euler’s equation is the one which is the simplified form of Navier stokes equation, which Euler’s equation is for inviscid fluid. Where viscosity can be set equal to 0 in Navier stokes equation, in order to obtain the Euler’s equation and we get Bernoulli’s equation starting from Euler’s equation, but that is a separate story.

So, inside the boundary, inside the hydrodynamic boundary layer the flow is viscous. So, the viscous transport of momentum as well as the convective transport of momentum, both will have to taken, will have to be taken into account. Similar to hydrodynamic boundary layer what we are discussing right now is thermal boundary layer. So, thermal boundary layer is the region in which the temperature changes, temperature varies with y from a value equal to T_s, all the way to the constant value of T_∞.

Here I have the velocity changing from 0 to V_∞, here the temperature changes from T_s which is the temperature of the substrate to infinity, beyond this point the velocity is constant, beyond this point the temperature is constant. So, there is a similarity between the thermal boundary layer, boundary layer and this is known as the hydrodynamic boundary layer. So, in most in most of the cases, the thickness of the hydrodynamic boundary layer which is generally denoted by δ and the thermal boundary layer which is denoted by δt they are not equal.

So, in one hand you have δt and over here you have δ. So, this δ the thickness of the boundary layer; obviously, as you can see from this figure is a function of x, here δt is also a function of x; however, δ and δt may not be equal.

(Refer Slide Time: 24:08)



So, we make it situation in which, if this is the solid plate; you have the thermal boundary layer and you have the hydrodynamic boundary layer. Or in you may have situations in which the thermal boundary layer would be below and the hydrodynamic boundary layer would be above.

So, let us call this is hydrodynamic boundary layer, and this is the thermal boundary layer so here it is going to be TBL and HBL on for some very special cases you discuss them in detail later on these two would coincide. So, when these two coincide here I have for hydrodynamic boundary layer, this is delta thermal boundary layer thickness at any location is given by is denoted by δt. So, in this case δ would be equal to δt. So, when we would see that this special condition only appears when Prandtl number which is defined as $Pr = \frac{\mu Cp}{k} = 1$

Now, why would that happen, let us let us expand this a little bit more, Prandtl number is

$$Pr = \frac{\mu Cp}{k} = \frac{\mu / \rho}{k / \rho Cp} = \frac{\nu}{\alpha}$$

I can write it as dividing both the numerator and the denominator by the ρ, the density I can write it as $k / \rho Cp$ and if you remember your fluid mechanics this μ / ρ is known as the kinematic viscosity and this $k / \rho Cp$ which you have just seen is denoted by α. So, this is known as the momentum diffusivity and this α is known as the thermal diffusivity. Both have units both α and ν have units of m^2/s, which is the same as the diffusion coefficient that you probably have heard of which is the diffusion coefficient of A and B this also have units of m^2/s.

So, another important observation which would not probably with related to heat transfer, but you are going to come across this many times is that conceptually α, ν and D_{AB} are all similar and all will have units of m^2/s. In one case this refers to momentum transfer, this refers to heat transfer and this refers to mass transfer. So, conceptually there is not much difference between these three transport processes, heat, mass and momentum transfer and they in there at some point of time would be the base on which the unified treatment of heat, mass and momentum transfer can be undertaken. That you are going to study in a separate course which is transport phenomena that looks at the fundamentals of all these transport processes.

But coming back to convective heat transfer or coming back to the nature, to the growth of these boundary layers, what we see is that the value of Prandtl number which is μ / ρ they simply tell us about momentum diffusivity and thermal diffusivity. So, the growth of these layers they are strongly dependent the TBL, the Thermal Boundary Layer would strongly depend on the thermal diffusivity. How fast, how easily the temperature front is getting into the moving fluid and the for the case of hydrodynamic boundary layer its α which is defining, how the hydrodynamic boundary layer is growing.

So, when numerically these two are equal both the HBL and the TBL would grow together, would grow at the same rate therefore, the value of Prandtl number equal to unity which appears only when momentum diffusivity and thermal diffusivity are equal. This is a special case which would let us take the value of the thickness of these two layers to be identical. So, this is the background which we are going to, which we are going to utilize in deriving some of the equations of convective heat transfer.

So, we understand here that conduction and convection both exist for the case of convection, but conduction can be a standalone process, which do not require the presence of convection. Conduction mostly, conduction when it happens inside a solid that is no question of any movement of the molecules, no net movement of the molecules and therefore, conduction is specified by 0 velocity or no velocity. Whereas, convection you have to have a velocity which could be imposed, that occurs in most of the industrial processes or it could be without the imposition of a velocity it is there because of the presence of a velocity.

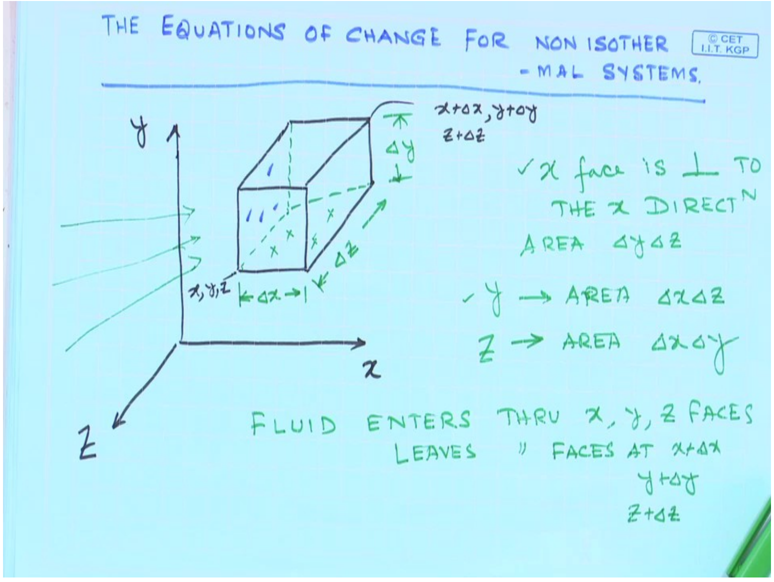
Because, in the presence of a temperature gradient which induces a difference in buoyancy a change in the value of the density are so therefore, have buoyant force would make the fluid in contact with the solid, hot solid rise and that is what is known as a natural convection.

Before we take this, slightly further and do of mathematical treatment of convection. Next another concept which I would like to introduce, a new I will follow it up with the in the next class with further details. In many of these cases the equations can be obtained, the governing equations can be obtained if you assume a small control volume through the faces of which heat mass and momentum can enter the control volume. So, I can assume in a, in a free space a cuboid of side, size $\Delta x \Delta y \Delta z$. So, through so; obviously, this cuboid will have 6 faces and truth is 6 faces the, let us say the mass is allowed to come and heat is allowed to come as well. So, I am going to first write or draw this cuboid and try to identify mentally, what are the process, what are the ways by which, let us see heat can enter into this control volume.

So, I am going to write the physics of flow into the control volume and the associated energy which comes into the control volume. So, this difference equation when I write and when I divide all sides by $\Delta x \Delta y \Delta z$ what I do is I convert the difference equation, which is a statement of the physics of the situation to a differential equation. In this differential equation can then be integrated with appropriate boundary conditions to obtain either the velocity profile or the temperature profile.

So, this kind of approach where a small cuboid is assumed in the flow space, the difference equation written convert it to differential equation and then solved is known as the shell momentum balance or shell heat balance or in the case of mass transfer it is known as the shell species balance. So, the first thing that one should do in order to derive all these equations or all these concepts is defined as a shell. So, let us try to define a shell and identify through the faces what is going to come in terms of energy into the space, into the space that I have defined over here.

(Refer Slide Time: 33:20)



THE EQUATIONS OF CHANGE FOR NON ISOTHER-MAL SYSTEMS.

✓ x face is ⊥ TO THE x DIRECT^N

AREA $\Delta y \Delta z$

✓ y → AREA $\Delta x \Delta z$

z → AREA $\Delta x \Delta y$

FLUID ENTERS THRU x, y, z FACES

LEAVES " FACES AT $x+\Delta x$

$y+\Delta y$

$z+\Delta z$

So, let us just draw, so this is my coordinate system and I am going to draw the shell. So, this is the one which I have, let us say this is my x, this is my y and this is the z. So, as you can clearly see this is Δx, this is Δz and this is Δy, this is the space which I have defined and I have flow of a liquid in all possible directions which approach in this.

So, this point is, the coordinate of this point is x, y and z and coordinate of this point is $x+\Delta x$, $y+\Delta y$ and $z+\Delta z$. So, I have 6 faces in this case, the face which is perpendicular to the x direction; that means, this face, which you do not see, this face is known as the x face. So, the x face is perpendicular to the x direction, its area as you can clearly see the area is going to be $\Delta y\Delta z$. Similarly, y face will have area, y face is the face below the bottom face which is this one, the y face will have area $\Delta x\ \Delta z$ and this z face which is the one which is see over here, this z face will have an area of $\Delta x\Delta y$.

So, what we would assume is that the fluid is going to enter through x, y. So, the fluid enters through x, y and z faces and leaves through faces at $x+\Delta x$, $y+\Delta y$ and $z+\Delta z$. So, this is what the fundamentals of shell moment, shell heat balance is. So, you have this shell of dimensions $\Delta x\Delta y\Delta z$ which is situated in a flow field and the flow is going to come, it would enter through all these faces in a when and leave through the other sides. That means, they what when it can come through x it leaves at $x+\Delta x$, y and $y+\Delta y$, z and $z+\Delta z$.

It is a three dimensional flow so, there is going to be components of velocity as V_x, V_y and V_z and the temperature is going to be a function of x, y and z as well. Whenever liquid enters into the control volume, it will carry with it some amount of energy and when it leaves it is going to carry some amount of energy. So, some amount of energy is coming into the control volume, some of it is going to live in the control volume, the coming to that of the energy to the control volume is through three faces, it is going to leave through the other three faces.

So, there could be as a result of this process a net amount of energy, which is added to the control volume this net amount of energy could also reduce. So, in that case I am going to simply use a minus sign, but let us assume that some amount of energy is added to the control volume, now this control volume can also do some work or some work maybe done on it.

So, that is possible since we are considering, taking into considerations all possibilities we should also consider that the control volume can do some work or some work can be done on it. So, the net amount of heat which you add and the amount of work that this system does or it is being done on the system, this sum total of this must be equal to the time rate of change of internal and kinetic energy of the system.

So, if you recall first law of thermodynamics what I have stated. So, far in terms of the control volume is nothing, but the statement of first law of thermodynamics for an open system. Where all effects are considered, the energy which comes with the flowing fluid, it will have a thermal energy component; it will also come with a velocity. So, there will be some kinetic energy component and an internal energy component, kinetic energy is because of the velocity of the fluid stream, internal energy is because of its temperature, whatever be its temperature.

So, we would like to write all those terms containing kinetic energy and internal energy through all the 6 faces, they are going to give me the net heat being net energy, both kinetic and internal being added to the control volume. We will also have to take into account whether the control volume does any work or some work is being done on the control volume, that is going to be another component of the difference equation. As a result of this the total energy content, internal and kinetic, total energy content of the control volume will change with time.

So, we are not restricting our self to steady state, we also allow the energy and the total energy can change which time inside the control volume. So, when I express in that terms, what I am stating is nothing, but the first law of thermodynamics, where all effects are considered and from this difference, difference equation when we think about, when we take all the appropriate terms into account for example, let us see work done on the system or by the system. So, what are the forces against to which what can be done, one obviously, is a body force for example, a gravity which acts on the entire volume of the control volume.

So, gravity is a body force which for against which the control volume may do some work, the other forces are not everywhere, not acting everywhere on the control volume, but acting on the surfaces, so surface forces will also have to be taken into account. So, what are the, for most what is the most common surface force is pressure, so the pressure force is acting on the control volume. So, these are two examples of the forces which can operate on the control volume, one is a body force the other is a surface force. So, we have to identify the most common body forces and surface forces plugged into the equation that I have just described, and then try to see mathematically; what is the end result of it.

So, the end result of it should give us an equation which is an energy equation, which should have embedded into it both conduction, convection the work done and as a result of which the time rate of change of temperature of the control volume. So, that would give me the complete energy equation, which I will be able to use for a specific application, cancelling the terms which are not relevant for the problem that we are dealing with and get to a simplified form of equation. The same way we have obtained the equation of conduction in the case of conductive heat transfer. So, I would stop here today, but what I have discussed is about conduction, the thermal boundary layer the velocity boundary layer and the concept of shell heat balance.

And while describing shell heat balance I have use the concept of first law of thermodynamics for an open system and the shell that I have defined of size $\Delta x \Delta y \Delta z$ which is fixed in space and through all the faces the fluid is coming with some internal energy and some kinetic energy. And some work is being done on the system or by the system and the work is being done either against gravity, which is the example of a body force or some pressure forces which are acting only on the surfaces of the define control volume. When I sum all of them together what I get is the total, the time rate of change of internal and kinetic energy is of the control volume of the shell that we have to just define.

So, we will write down the terms in the next class and see how beautifully this concept would result in a compact differential equation for temperature and this for the spatial; that means, x, y, z variation of temperature, as well as the temporal variation of temperature, that means how the temperature changes with time. So, my aim is to obtain an expression of t as a function of x, y, z and time that is what I would like to get out of this energy equation. And once I have that, then I am going to transform this equation for the case of convective heat transfer to see whether or not I get a compact expression for h, the convective heat transfer coefficient the engineering parameter of interest that engineers would like to find out before designing any equipments.

So, starting from fundamentals through some, little bit of mathematics I would like to get a differential equation, from the differential equation and expression of each which can be used by practicing engineers. So, that is the whole chain which I would like to cover in the, in some of the future classes.

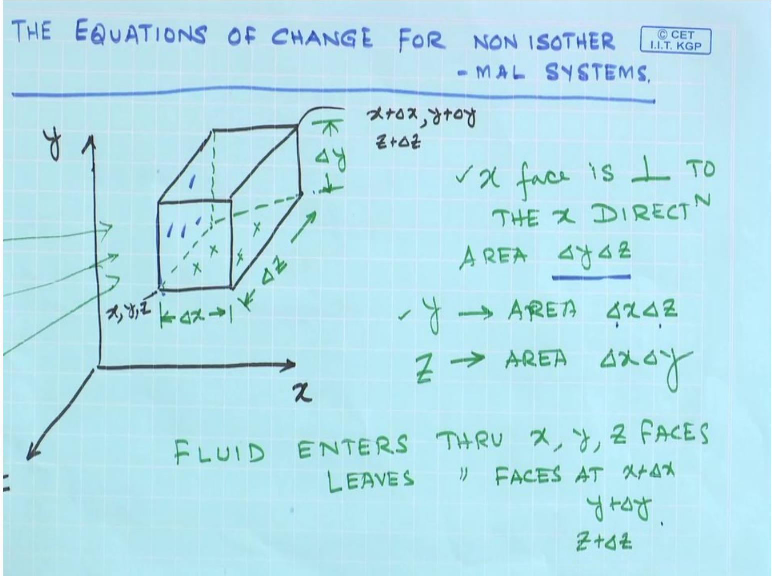
Heat Transfer
Prof. Sunando Dasgupta
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 17
Equations of Change for Non-isothermal Systems

I would like to introduce a concept which we started discussing in the last class, which is called the shell balance. Now, this shell balance can be of momentum, it can be of energy it can also be of a species. When we do a shell momentum balance we get a governing equation that describes the change in velocity as a function of x, y, z and time. When we write the shell heat balance we should get what is known as the equation of energy. Similarly, if I write it for a species, which is let us say reacting with another species in a flowing fluid field, then we would get the species conservation equation which is going to be very important in mass transfer. Let us try to concentrate on how we can write a shell heat balance, the trick is to define a shell of let us say some size del x, del y, del z which is fixed in space.

So, a cuboid shape of size del x, del y, del z we have 6 faces: one is the x face which is perpendicular to the x direction, one is the y face which is perpendicular to the y direction and the other is going to be that z face which is perpendicular to the z direction. Each of these faces will have areas associated with them and through these areas heat energy can come into the control volume. Because of which the internal energy of the box would change. When we talk about energy will we are not going to talk only about the internal energy, we also must consider the kinetic energy. So, a fluid may come through the x face with certain velocity and therefore, certain kinetic energy, at a temperature which is different form the temperature of the fluid contained in the box. Therefore, the entering fluid will have some internal energy and some kinetic energy associated with it. So, it will enter the x face and will leave the x plus delta x face, similarly it would come through y, leave at y plus delta y, come at z and leave at z plus delta z.

(Refer Slide Time: 05:44)



THE EQUATIONS OF CHANGE FOR NON ISOTHERMAL SYSTEMS.

✓ x face is ⊥ TO THE x DIRECT^N

AREA $\Delta y \Delta z$

✓ $y \rightarrow$ AREA $\Delta x \Delta z$

$z \rightarrow$ AREA $\Delta x \Delta y$

FLUID ENTERS THRU x, y, z FACES

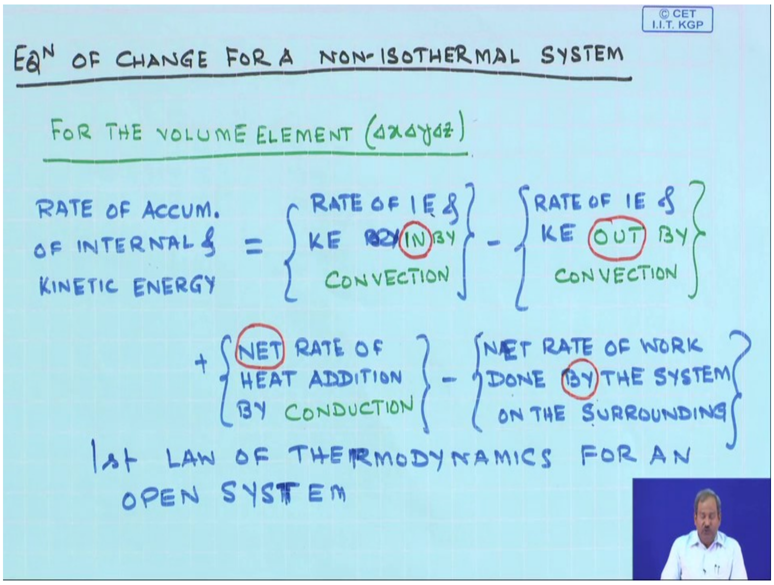
LEAVES " FACES AT $x+\Delta x$

$y+\Delta y$.

$z+\Delta z$

Let us once again go through the equation of change for a non-isothermal system, here I have drawn this box which is del x, del z and del y. These are the x, y and z directions and the x face which is this one is perpendicular to the x direction and therefore, its area is $\Delta y \Delta z$. Similarly, the y face which is perpendicular to the y direction would should have area of $\Delta x \Delta z$; and the z face would have an area of $\Delta x \Delta y$. We are going to now write what would be the form of the energy equation for a system of size del x, del y, del z which is which is fixed in space and this is our coordinate system.

(Refer Slide Time: 07:10)



EQ^N OF CHANGE FOR A NON-ISOTHERMAL SYSTEM

FOR THE VOLUME ELEMENT ($\Delta x \Delta y \Delta z$)

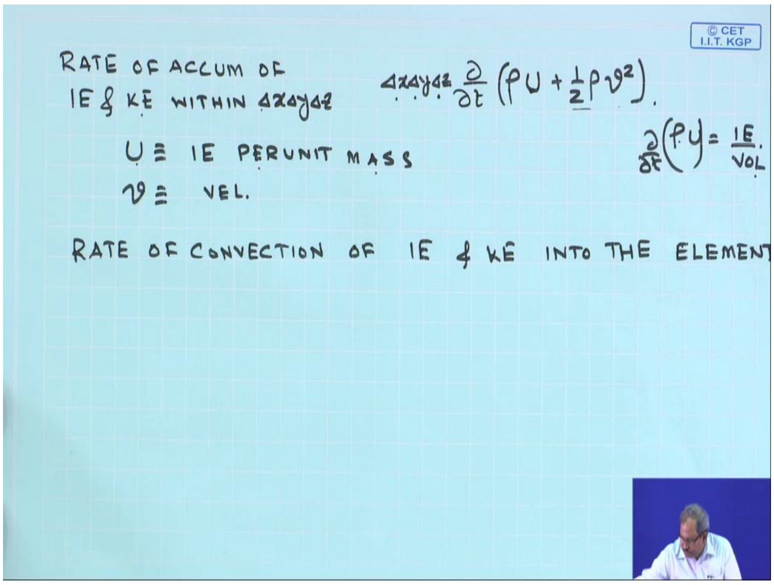
RATE OF ACCUM. OF INTERNAL & KINETIC ENERGY = {RATE OF IE & KE ~~BY~~ IN BY CONVECTION} − {RATE OF IE & KE OUT BY CONVECTION} + {NET RATE OF HEAT ADDITION BY CONDUCTION} − {NET RATE OF WORK DONE BY THE SYSTEM ON THE SURROUNDING}

1st LAW OF THERMODYNAMICS FOR AN OPEN SYSTEM

So, let us see how the equation of change would look like for a non-isothermal system. When there is a flow, some amount of energy comes with the flow and when I talk about energy I speak about both the internal and the kinetic energy. So, some internal and kinetic energy can come into this volume element by convection and it is going to go out again by convection, from the x plus del x, y plus del y and z plus delta z faces. Let us say I have a temperature difference, which exists in the x direction; then obviously, I am going to have some flow of heat through conduction through this x face. So, a difference in temperature either in x or in y or in z even if the fluid is still, would give rise to conductive heat transfer. So, if there is a temperature difference, there is going to be a conductive heat transfer. I have combined the terms together to write it in the form of net rate of heat addition to the volume element by conduction. So, this takes care of all the heat that comes to the system by conduction or by convection. However, there is one missing term that I should consider at this point, that is the rate of work done by the system on the surrounding; and hence, the total energy of the system should reduce, and that is why we have a minus sign. Had this been a case of work being done on the system, then this sign should be positive. So, what I have written over here is nothing, but the first law of thermodynamics, for an x and since I am allowing fluid to enter and leave, this must be for an open system.

Now, from this generalized energy equation one should be able to deduct the commonly available equation for kinetic energy of a system. And therefore, what you would left out with is the energy equation where we are only considering internal energy which is manifested by a change in temperature.

(Refer Slide Time: 12:41)



RATE OF ACCUM OF
IE & KE WITHIN $\Delta x \Delta y \Delta z$ $\quad \Delta x \Delta y \Delta z \frac{\partial}{\partial t}\left(\rho U + \frac{1}{2}\rho v^2\right).$

$U \equiv$ IE PER UNIT MASS $\quad \frac{\partial}{\partial t}(\rho U) = \frac{\text{IE}}{\text{VOL}}.$

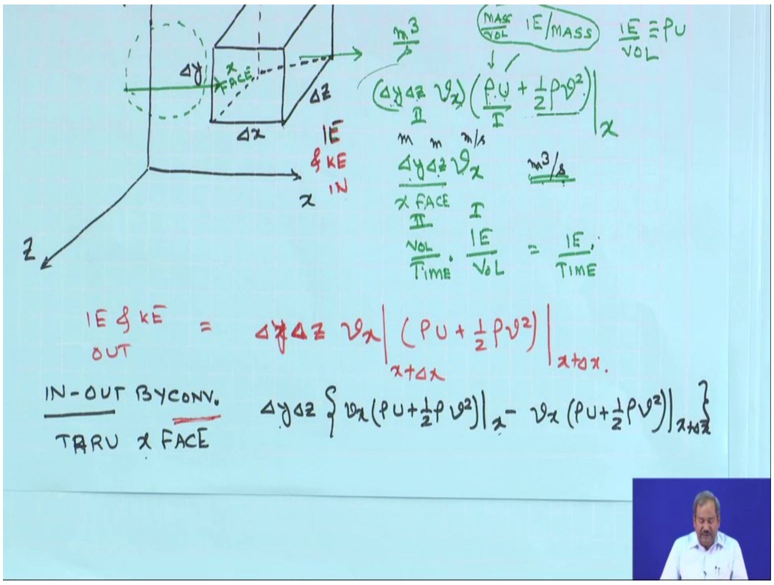
$v \equiv$ VEL.

RATE OF CONVECTION OF IE & KE INTO THE ELEMENT

So, first of all let us see what is going to be the rate of accumulation of internal energy and kinetic energy within the system, which is defined as del x del y del z. So, this must be $\frac{\partial}{\partial t}\left(\rho U + \frac{1}{2}\rho v^2\right)$ where, U is the internal energy per unit mass and v is the velocity. This internal energy per unit mass, here I have multiplied it with rho. So, this becomes internal energy per unit volume. So, if I would like to find out what is the total rate of accumulation of internal and kinetic energy within x, y, z this must be multiplied by del x del y del z, which makes it the rate of change of internal energy for a system, whose dimensions are del x del y and del z. As long as my U is defined as internal energy per unit mass, rho is the density.

Therefore, $\left(\frac{\partial}{\partial t}(\rho U)\right)(\Delta x \Delta y \Delta z)$ would simply give us the rate of accumulation of internal energy within del x del y del z and similarly the same logic will also be applied to $\frac{1}{2}\rho v^2$. The next one is, I am going to write would be the convection of internal energy and kinetic energy into the element.

(Refer Slide Time: 15:26)



In this figure, some amount of let us, this is my x face, which has an area of $\Delta y \cdot \Delta z$, and some amount of fluid is going to enter through this x face. Let us say, this velocity with which it comes into the control volume is v_x. So, when you multiply $\frac{1}{2}\rho v^2$, with $(\Delta y \cdot \Delta z)\cdot v_x$ it is going to give you the kinetic energy per unit time. So, together this whole term gives is evaluated at x on this face. So, it comes in through the x face and goes out through the face at x plus del x.

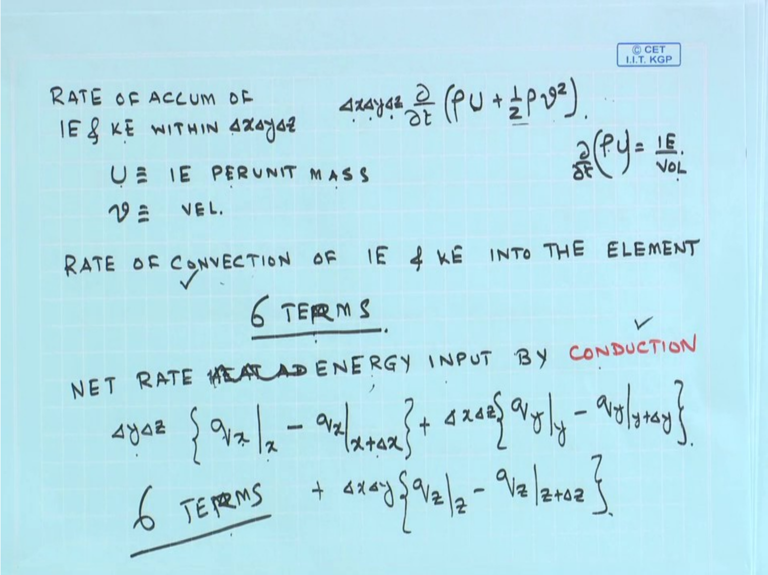
So, the entire thing is going to be at $(\Delta y \cdot \Delta z)\cdot\left(\rho U + \frac{1}{2}\rho v^2\right)\cdot v_x$, all evaluated at x plus delta x. So, the amount of energy, internal and kinetic energy in due to convection. So, when I consider the net energy balance due to convection, the equation would be:

$$\Delta y \cdot \Delta z \left\{ \underbrace{\left(\rho U + \frac{1}{2}\rho v^2\right)\cdot v_x}_{at\ x} - \underbrace{\left(\rho U + \frac{1}{2}\rho v^2\right)\cdot v_x}_{at\ x+\Delta x} \right\}$$

So, this is what the expression for in minus out by convection through the x face would look like. Now, I can write the same thing for the y face, the only thing which will be different here is the y face has an area equal to $\Delta x \cdot \Delta z$ and since we are talking about y then the v_x must be replaced with v_y and instead of evaluating it at x, it is going to be evaluated at y and y plus delta y, similarly for z face this area is going to be $\Delta x \cdot \Delta y$, v_x is to be replaced by v_z and everything else will remain same.

So, in total to consider the convective flow of internal and kinetic energy into the volume element, del x del y del z, I will have 6 terms. Two terms each for x, y, and z face would give me total of 6 terms that would signify what is the total amount of heat which comes into the system by convection.

(Refer Slide Time: 23:01)



RATE OF ACCUM OF IE & KE WITHIN $\Delta x \Delta y \Delta z$ $\Delta x \Delta y \Delta z \frac{\partial}{\partial t}\left(\rho U + \frac{1}{2}\rho v^2\right)$.

$U \equiv$ IE PER UNIT MASS

$v \equiv$ VEL.

$\frac{\partial}{\partial t}(\rho U) = \frac{IE}{VOL}$.

RATE OF CONVECTION OF IE & KE INTO THE ELEMENT

6 TERMS.

NET RATE ~~HEAT AD~~ ENERGY INPUT BY CONDUCTION

$$\Delta y \Delta z \left\{ q_x\big|_x - q_x\big|_{x+\Delta x} \right\} + \Delta x \Delta z \left\{ q_y\big|_y - q_y\big|_{y+\Delta y} \right\}.$$

$$+ \Delta x \Delta y \left\{ q_z\big|_z - q_z\big|_{z+\Delta z} \right\}.$$

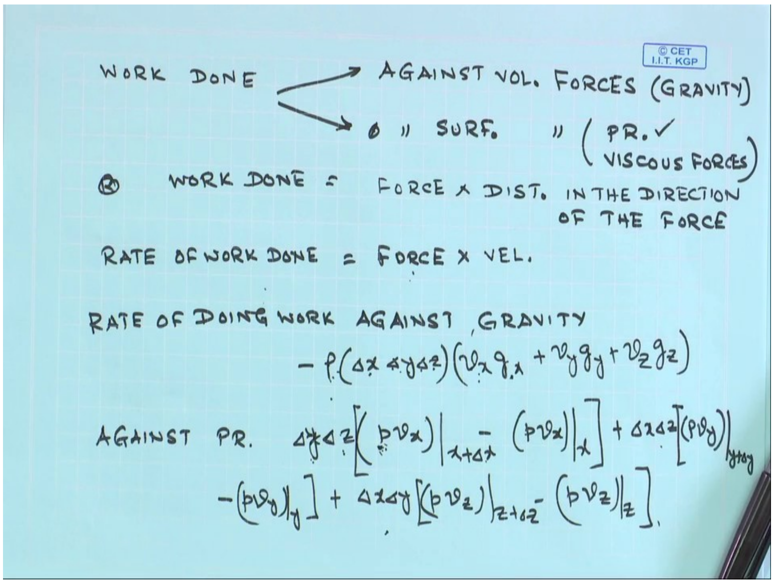
6 TERMS

As I have explained before, we must think about net rate of heat addition by conduction, I would not say heat here, energy because I also have the kinetic energy to take care of, net rate of energy input by conduction. And here I am going to express it in terms of the component of heat flux in the x direction.

So, q_x is the heat in per unit area per unit time. So, I must multiply it with the appropriate area since it is x face this must be equal to $\Delta y \cdot \Delta z$. So, this denotes the heat that comes to the control volume through the x face and the one that goes out through the x plus delta x face. So, these two terms together one at x and one at x plus del x multiplied by $\Delta y \cdot \Delta z$, together they tell us about the net rate of heat addition by conduction through the x face.

Similarly, I am going to have the y face which will have $\Delta x\Delta z$, the area and the heat flux is are going to be q_y, at y minus q_y at y plus delta y and for the z face it is going to be $\Delta x\Delta y$ times q_z, evaluated at z minus q_z at z plus delta z. So, these 6 terms again, 6 terms for each of the faces would tell me about the net energy input by conduction and so I have taken care of the convection and I have taken care of conduction. So, what is left is work done by the system on the surrounding.

(Refer Slide Time: 25:41)



WORK DONE → AGAINST VOL. FORCES (GRAVITY)

→ " SURF. " (PR. ✓ VISCOUS FORCES)

WORK DONE = FORCE × DIST. IN THE DIRECTION OF THE FORCE

RATE OF WORK DONE = FORCE × VEL.

RATE OF DOING WORK AGAINST GRAVITY

$$-\rho(\Delta x\Delta y\Delta z)(v_x g_x + v_y g_y + v_z g_z)$$

AGAINST PR. $\Delta y\Delta z\left[(pv_x)\big|_{x+\Delta x} - (pv_x)\big|_x\right] + \Delta x\Delta z\left[(pv_y)\big|_{y+\Delta y} - (pv_y)\big|_y\right] + \Delta x\Delta y\left[(pv_z)\big|_{z+\Delta z} - (pv_z)\big|_z\right]$

© CET I.I.T. KGP

Now, work done as we all know it simply, it can be against volumetric forces, volumetric forces that which are acting on the entire volume of the volume element. The common example would be gravity and the second one is against surface forces, surface forces which could be against pressure, which could be against viscous forces. So, these two are again the common examples of surface forces.

Now, I am not interested in work done, because work done is, force times distance in the direction of the force. So, what would be rate of work done that is the time rate of work done, it would be force times distance by time. So, the work done is given by:

$\rho(\Delta x\Delta y\Delta z)(v_x g_x + v_y g_y + v_z g_z)$. We know that, distance by time; obviously, would give you the velocity. So, this is what you are going to get for the rate of work done. So, rate of work done would simply be expressed as force times velocity. So, let us quickly write the expression for the force against gravity forces, so rate of doing work against gravity would simply be equal to minus since, it is against work is done against gravity.

So, v x times g x velocity and this is acceleration due to gravity plus v y g y plus v z g z. So, when you, when you see this equation you would be able to see that it is the force in the x direction which is v x, del x del y del z times rho and this is this is multiplied by g x. So, this totally gives you the rho del x del y del z times g x is the force, because this is mass, this is mass per unit volume this is volume, and this is the acceleration. So, this gives you the force and force multiplied by the velocity in the appropriate direction. So, v x, v y, and v z would give you the rate of work, rate of doing work against gravity.

So, again what is going to be the form for against pressure, it should be the area on which let u say the x face del z times P v x, evaluated at x plus delta x minus P v x at x. So, this is one term, plus del x del z, p v y at y plus del y minus p v y at y, this is going to be the second term plus del x del y, p v z times z plus del z minus p v z. Look at these terms one more time and see what the mean, pressure is force per unit area. Whatever be the pressure at x plus del x is multiplied by the appropriate area which is del y del z, to give to give us the force in the x direction acting on the control volume, acting on the volume element at x plus del x and we understand that the rate of work done is force times velocity. So, the pressure-work is given by:

$$\left(\Delta y \Delta z\right)\left(\left.\underset{\sim}{P}\underset{\sim}{v}_x\right|_{x} - \left.\underset{\sim}{P}\underset{\sim}{v}_x\right|_{x+\Delta x}\right) + \left(\Delta x \Delta z\right)\left(\left.\underset{\sim}{P}\underset{\sim}{v}_y\right|_{y} - \left.\underset{\sim}{P}\underset{\sim}{v}_y\right|_{y+\Delta y}\right) + \left(\Delta x \Delta y\right)\left(\left.\underset{\sim}{P}\underset{\sim}{v}_z\right|_{z} - \left.\underset{\sim}{P}\underset{\sim}{v}_z\right|_{z+\Delta z}\right)$$

So, these 6 terms together would give us the work done against the pressure forces by the volume element del x del y del z. What is remaining here is the work done against viscous forces, now work done against viscous forces this I am going to neglect for the time-being because work done against viscous forces is something like solid friction. So, what happens when you work against friction forces, you are pulling an object over a rough surface. So, you have to overcome the frictional forces exerted by the rough surface, as a result of which there is going to be heat generation in it and any work that you do that in order to make that block move over a rough surface, is going to be converted into heat and it will change the energy of the of the system. Similarly, when fluid flows specially at high speed through a small duct, there is going to be tremendous velocity gradient which is present.

So, let us say I have a jet which very thin, and the fluid is coming at a very high velocity. So, the velocity is large and if the velocity is large and the gap is small, then the velocity gradient would be very large, and we understand that the viscous force is related to velocity gradient. The shear stress is mu times velocity gradient. So, if the velocity gradient is large or if the viscosity is large, in that case you will have a strong force that you need to overcome in order to make the fluid flow through that thin conduit at a very high velocity. If that happens then you do substantial work against the viscous forces and whenever you do that kind of work against viscous forces the temperature will increase. And that increase in temperature which is obtained at the expense of work done by the system must be considered for any form of energy equation.

However, this is only relevant in some special situations, we do not get the heat generation due to viscosity in many of the practical problems. As you would see it requires high velocity gradient and very high viscosity. So, what are the places in which they become relevant, when a rocket reentered earth's atmosphere its velocity is very large, the atmosphere is still, but the rocket is coming down with a very high velocity. So, near the boundary layer, formed close to the rocket, the velocity changes from that of the rocket which is very large, to velocity equal to 0 which is the velocity of the atmosphere. So, this thinness of the boundary layer and the very high speed of the rocket at reentry would ensure that the frictional heat generated is tremendous. And that is why you would see that the rocket comes almost like a red-hot object and there has to be special protective arrangements to ensure the safety of the astronauts inside the rocket.

So, that is an extreme example, in some cases viscous polymer is extruded by making it flow through a very thin gap, if that is the case then the viscosity is high, the velocity is large as you would like to have higher throughput of the polymer when you are making a sheet out of it. So, the velocity combined with the high viscosity of the polymer ensures that you cannot neglect viscous dissipation. However, we will neglect viscous heat dissipation, wherever the viscous heat dissipation is irrelevant; but I will tell you that how to incorporate additional terms into the energy equation which would take into account the viscous heat generation.

So, if you look at your textbook and look at the full form of the energy equation, you would see there are a bunch of terms which have which are multiplied with mu. So, the easiest way to identify which term of your energy equation in your text relates to be, relate to viscous heat generation, is to look for terms containing mu. If in your problem the viscous heat generation is negligible drop the entire set of terms containing mu and what you would have is the energy equation that we are going to use for most of the realistic applications. So, since those terms are complicated, I am dropping them for the time being, but making you aware that in some special situations you need to add them to ensure that your energy equation is complete.

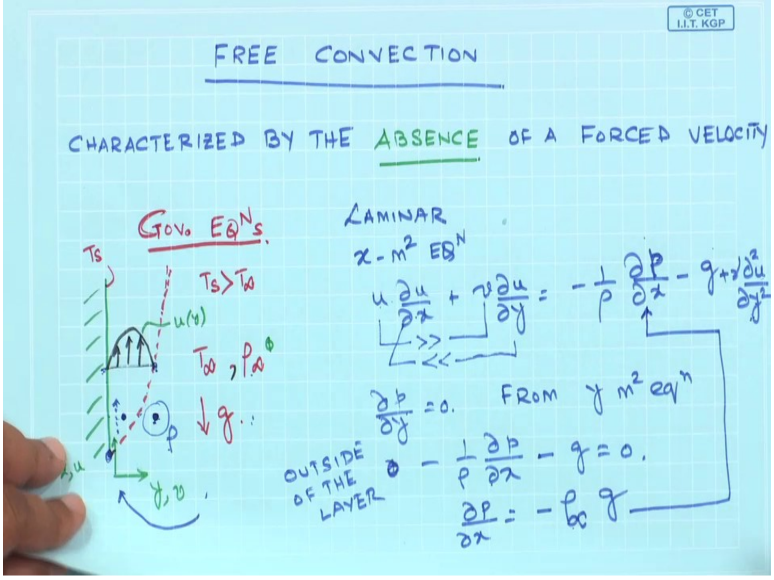
Heat Transfer
Prof. Sunando Dasgupta
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 37
Free Convection

In previous classes, we have seen the relations and correlations for external flow as well as for internal flow. In all these cases there was an imposed pressure gradient specially for internal flow which drives the fluid through the tube. There are several locations in which there is no externally imposed flow on the system even though a flow will automatically start because of a difference in buoyancy. So, this buoyancy difference driven flow will lead to convection as well which is known as the free convection or natural convection. This is a very important part of the overall transport process around many structures, buildings and surfaces. So, whenever there is an absence of a forcing velocity, the heat transfer under that situation which is caused by density difference due to temperature is termed as free convection or natural convection. The applications of natural convection as I mentioned are many: they appear in atmospheric process to semi semiconductor chips, when a semiconductor chip is cooled in absence of a fan which forces the fluid to move over it, the way it loses heat is natural convection. So, from a very small length scale to a very large length scale, we will have natural convective heat transfer under many situations. It is therefore important to know the physics of natural convection, what are the relations and correlations which are to be used in order to obtain the heat flux or the heat transfer coefficient in natural convection. And just to have an idea of the order of magnitude value of the heat transfer coefficient that one would encounter in natural convection process.

So, let us look at the development of the governing equation and see what additional information we can obtain by simply writing the equation of energy, equation of motion, and equation of continuity and see what the similarity parameter would be, or which dimensionless group would automatically appear in those equations. It is to be noted that any relation of heat transfer coefficient will therefore, be a function of Nusselt number besides Reynolds and Prandtl. We would observe that some other number may also appear for the case of natural convection and the point to start this analysis is by using the fundamental equations - the energy, the motion, and the continuity equation.

(Refer Slide Time: 03:18)



© CET I.I.T. KGP

FREE CONVECTION

CHARACTERIZED BY THE ABSENCE OF A FORCED VELOCITY

GOV. EQ^N^S.

LAMINAR

x - m^2 EQ^N

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} - g + \nu\frac{\partial^2 u}{\partial y^2}$$

$>>$

$<<$

$\frac{\partial p}{\partial y} = 0$. FROM y m^2 eq^n

OUTSIDE OF THE LAYER

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} - g = 0.$$

$$\frac{\partial p}{\partial x} = -\rho_\infty g$$

So, let us look at the figure over here which shows that I have a plate which is in contact with air. The plate temperature is T_S and the temperature of the surrounding air at a point far from the plate is T_∞ and the corresponding density of air is ρ_∞. The gravity is acting vertically downwards; and we will assume that the temperature of the plate is more than the temperature of the air. Therefore, the temperature of the fluid near the wall will increase, its density will decrease, and it starts to rise. As it rises, this fluid is going to be replaced by the cooler fluid from beneath. So, you have a cooler fluid from over here and then the process would start once again. So, the hot fluid near the wall rises towards the top to be replaced by cold fluid from the ambient. This is what is known as free convection and the free convection is classified based on whether the flow is bounded by a surface or if it is a free boundary flow.

So, let us write the equation; let us assume that the flow due to the density difference in forced flow inside the boundary layer is laminar. But however, it is also possible that will have turbulence at some point at a larger value of x. So, this is x = 0, y = 0; that is the origin that we have and the velocity in the x-direction is u, the velocity in the y direction is v and we understand that the flow outside of this thin layer is essentially stationary. So, if I draw the velocity profile of the air which is rising close to the plate, it is going to be 0 at x = 0 and y = 0; due to the no-slip and no penetration boundary conditions. As we progress further, we would observe that the profile would be somewhat parabolic with the maximum velocity occurring somewhere in between. We understand that v is going to be very small, i.e. the velocity in the y-direction is going to be very small as compare to the velocity in the x direction. Therefore, the x momentum equation is simply going to be $u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial x}-g_y+\vartheta\frac{\partial^2 u}{\partial y^2}$. Note that I have neglected the viscous transport of momentum in the x direction as the velocity gradient in the x direction is very small as compared to the velocity gradient in the y direction due to the thinness of the region where this flow is taking place. However, u is large as compared to v, but $\frac{\partial u}{\partial y}$ is very large as compared to $\frac{\partial u}{\partial x}$.

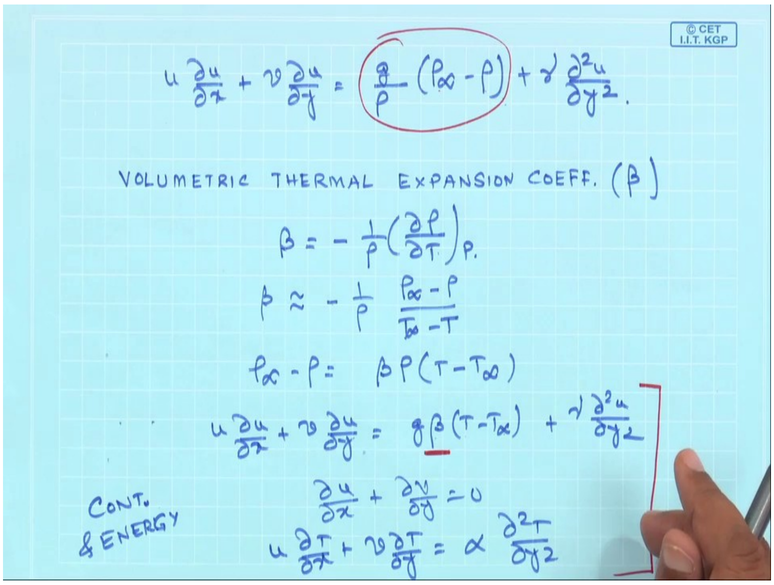
So, none of the terms on the left-hand side could be neglected. However, we also know that if I write the y component of the equation of motion, since there is no force in the y direction, the gravity is acting downwards. So, y momentum equation would give you $\frac{\partial P}{\partial x}=0$. The pressure gradient at any point inside the boundary layer must be equal to the pressure gradient in the region outside the boundary layer. So, the pressure gradient at these 2 points would be identical since $\frac{\partial P}{\partial x}=0$ and P does not vary in the y direction. If that is the case, then in this region, u is 0. There is no velocity in the x direction, no velocity in the y direction. The effect of gravity would be there, but the velocity variation with y is also not going to be present.

So, the equation that I have written over here, the x component of the momentum equation; if I write it for a region outside of the thermal boundary layer or outside of this layer, then all these terms would be 0 except $-\frac{1}{\rho}\frac{\partial P}{\partial x}-g_y=0$. So, that is the form of the equation if I write it outside of the layer. Therefore, $\frac{\partial P}{\partial x}=-\rho g_y$; and the rho is essentially ρ_∞.

So, if I put this equation back in here, then the left-hand side will remain the same, in the right-hand side, these 2 terms can therefore be clubbed together to give the equation as

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{g_y}{\rho}\left(\rho_\infty-\rho\right)+\vartheta\frac{\partial^2 u}{\partial y^2}.$$

(Refer Slide Time: 09:43)



$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\boxed{\frac{g}{\rho}\left(\rho_\infty-\rho\right)}+\nu\frac{\partial^2 u}{\partial y^2}.$$

VOLUMETRIC THERMAL EXPANSION COEFF. (β)

$$\beta=-\frac{1}{\rho}\left(\frac{\partial\rho}{\partial T}\right)_P$$

$$\beta\approx-\frac{1}{\rho}\,\frac{\rho_\infty-\rho}{T_\infty-T}$$

$$\rho_\infty-\rho=\beta\rho\left(T-T_\infty\right)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g\underline{\beta}\left(T-T_\infty\right)+\nu\frac{\partial^2 u}{\partial y^2}$$

CONT. & ENERGY

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}$$

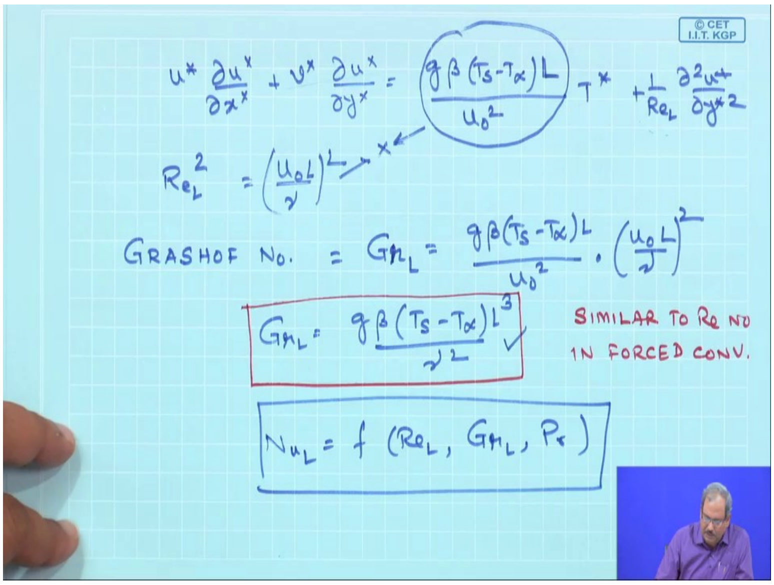
Now, this is the term which is the force body force acting on the fluid under natural convection condition. To express this in a more convenient form, we introduce a thermodynamic quantity which is the volumetric thermal expansion coefficient. The volumetric thermal expansion coefficient which is termed as β, and $\beta=-\frac{1}{\rho}\left(\frac{\partial\rho}{\partial T}\right)_P$;

so, β can also be approximated as $-\frac{1}{\rho}\left(\frac{\rho_\infty-\rho}{T_\infty-T}\right)$. Therefore, this $\rho_\infty-\rho$ can be written as $\beta\rho\left(T-T_\infty\right)$. Therefore, the x-direction momentum equation can be written as: $u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g_y\beta\left(T-T_\infty\right)+\vartheta\frac{\partial^2 u}{\partial y^2}$. So, this is the equation of motion where we have introduced a thermodynamic parameter β which is known for most of the gases. The analysis was undertaken because it is better to express the equation in terms of the delta

T rather than in terms of delta rho by introducing the concept of volumetric thermal expansion coefficient. The continuity equation can be written as: $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$; and the energy equation can be represented as: $u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}$; where α is the thermal diffusivity.

So, these 3 equations are to be solved simultaneously in order to obtain what is known as the thermal coefficients for the relations for natural convection. One point I would like to make here is that, the next step involves non-dimensionalizing the equation. As we have done before and in order to non-dimensionalize this, we need to define a length scale which is straight forward. Because, the length scale would most likely be the length of the plate along which this rise of the liquid is taking place. I also must bring in some reference velocity which is yet to be decided. I am simply going to write the dimensionless form of the equation without going through the all the steps, what I would suggest you is to look at Incropera and Dewitt, the textbook of this course for the derivations from the dimensional equation to the dimensionless equation, but those steps are merely substituting the variables in terms of the dimensionless variables and no new physical concepts are involved. So, we will skip those steps and only write the dimensionless form of the equation which would give us more insight into the natural convection process.

(Refer Slide Time: 14:26)



$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = \boxed{\frac{g\beta(T_s - T_\infty)L}{u_0^2}}\, T^* + \frac{1}{Re_L}\frac{\partial^2 u^*}{\partial y^{*2}}$$

$$Re_L^2 = \left(\frac{u_0 L}{\nu}\right)^2 \rightarrow \times$$

GRASHOF No. $= Gr_L = \frac{g\beta(T_s - T_\infty)L}{u_0^2} \cdot \left(\frac{u_0 L}{\nu}\right)^2$

$$\boxed{Gr_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}} \checkmark$$

SIMILAR TO Re NO
IN FORCED CONV.

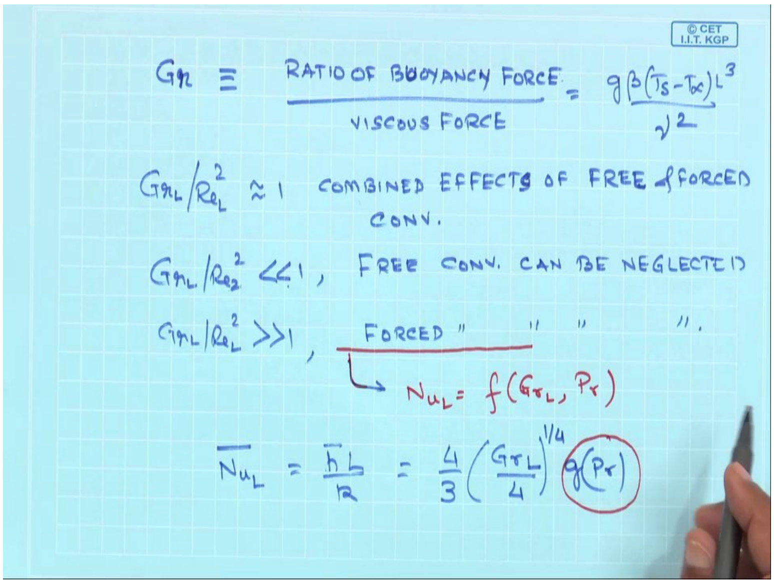
$$\boxed{Nu_L = f(Re_L, Gr_L, Pr)}$$

Therefore, the dimensionless form of the equation is written as: $u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = \frac{g_y \beta \left(T_s - T_\infty\right) L}{u_0^2} + \frac{1}{\mathrm{Re}_L} \frac{\partial^2 u^*}{\partial y^{*2}}$; where L is the length scale, u_0 is the velocity difference that is yet to be determined (reference velocity). Just see this equation and compare this equation mentally. What would we have obtained if this is a case where there is no temperature gradient present in the system? So, if there is no pressure gradient present and no temperature gradient present in the system then, this equation of motion would simply revert to the equation that we already know the first term on the right-hand side would be 0 since $T_s = T_\infty$. So, these 2 terms will remain, and this term will remain as well, and you can clearly see that the similarity parameter. For such a case, as you have seen countless times before would be the Reynolds number. So, for a situation in which there is no temperature difference, natural convection does not take place and the equation of motion will have the known similarity parameter as Reynolds number only. However, in presence of a temperature difference between the plate and the surrounding air I have an additional extra term as the first term on the right-hand side of the of the momentum equation. So, anything that I define will not only contain the similarity parameter Reynolds number, but it should also contain something which will consist of contribution from the first term and again the contribution from the first term is nothing but the contribution from natural convection.

So, let us work a little bit on that term to see if any additional dimensionless terms come out of the first term on the right-hand side. But in order to do that, if you look at your class notes once again you have a u_0^2 present in the equation. We know that term u_0 is a velocity that we have not yet defined. So, we need to get rid of this u_0^2 in some way. Let us multiply both sides of the equation by Re^2_L, which is nothing, but the Reynolds number based on the entire length which would be $\left(\frac{u_0 L}{\vartheta}\right)^2$. Therefore, what you would get is a new dimensionless number which is known as Grashof number, which is denoted by Gr, and the Grashof number based on the length is defined as: $\frac{g_y \beta\left(T_s - T_\infty\right) L^3}{\vartheta^2}$. The Grashof number has the same significance as Reynolds number in forced convection. We know that for forced convection, the Nusselt number is a function of Reynolds number and Prandtl number. So, whatever role Reynolds number played in the case of forced convection, the same role will be played by this Grashof number in the case of natural convection or free convection. So, the Grashof number can simply be expressed as the flow due the ratio of 2 forces; the viscous forces in the denominator and the temperature induced forces in the numerator.

So, the Nusselt number for natural convection should be a function of Reynolds number a function of Prandtl number and a function of Grashof number. So, unlike forced convection, we will have an additional term present in any relation or correlation of natural convection which is Grashof number. So, let us look at Grashof number in a bit more detail.

(Refer Slide Time: 20:20)



$$Gr \equiv \frac{\text{RATIO OF BUOYANCY FORCE}}{\text{VISCOUS FORCE}} = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}$$

$Gr_L/Re_L^2 \approx 1$ COMBINED EFFECTS OF FREE & FORCED CONV.

$Gr_L/Re_L^2 \ll 1$, FREE CONV. CAN BE NEGLECTED

$Gr_L/Re_L^2 \gg 1$, FORCED " " " ".

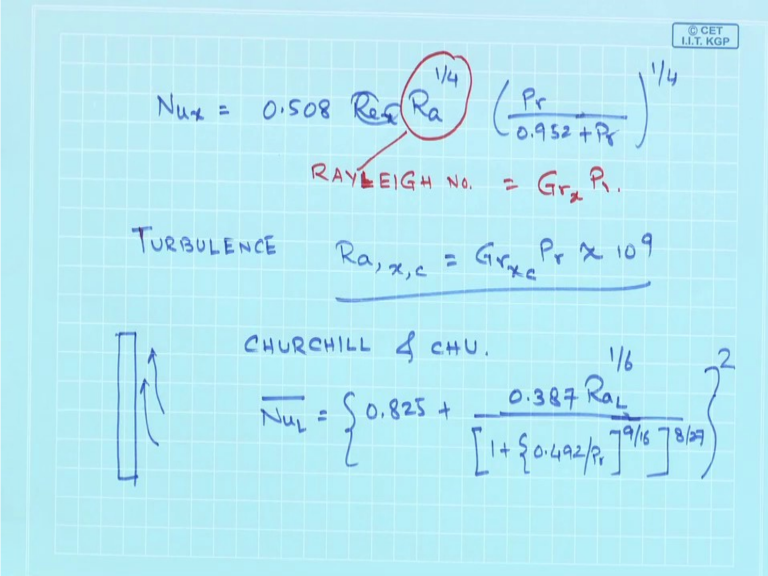
$\to Nu_L = f(Gr_L, Pr)$

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = \frac{4}{3}\left(\frac{Gr_L}{4}\right)^{1/4} \boxed{g(Pr)}$$

In natural convection, additionally, this Grashof number if you look at the definition of Grashof number it is nothing but the ratio of buoyancy forces to the viscous force. So, if $\frac{Gr_L}{\mathrm{Re}_L^2} \simeq 1$, the combined effects of natural and forced convection are to be considered. On the other hand, if $\frac{Gr_L}{\mathrm{Re}_L^2} << 1$ then free convection can be neglected. Conversely, if $\frac{Gr_L}{\mathrm{Re}_L^2} >> 1$ then the forced convection effect can be neglected. For such a case, the Nusselt number would simply be a function of Grashof number and Prandtl number. Many relations and correlations are available for free convections; I will simply give you some examples of those relations. For example, the average value of Nusselt number in natural convection can be written as: $\overline{Nu}_L = \frac{\bar{h}L}{k} = \frac{4}{3}\left(\frac{Gr_L}{4}\right)^{1/4} g\left(\mathrm{Pr}\right)$. This g(Pr) is a function of Prandtl number the values of which are provided in the text. These correlations are quite complicated, and of course I do not want you to memorize and it is not also possible to remember all these relations or correlations, but you should know when to use which relation. So, in any questions in any exam, I am going to provide all these relations to you, and you must pick the right relation or correlation citing justification and then use it to solve the problems.

I am going to give you some more examples of these relations; so, some other relations for natural convection are: $Nu_x = Ra^{\frac{1}{4}}\left(\frac{\Pr}{0.952+\Pr}\right)^{\frac{1}{4}}$; where, Ra is the Rayleigh number, and it is simply Grashof number at x times Prandtl number ($Ra = Gr_x \Pr$).

(Refer Slide Time: 25:10)



© CET I.I.T. KGP

$$Nu_x = 0.508\ \cancel{Re_x}\ \boxed{Ra^{1/4}} \left(\frac{P_r}{0.952+P_r}\right)^{1/4}$$

RAYLEIGH NO. $= Gr_x P_r$.

TURBULENCE $\underline{Ra_{,x,c} = Gr_{x_c} P_r \approx 10^9}$

CHURCHILL & CHU.

$$\overline{Nu_L} = \left\{0.825 + \frac{0.387\,Ra_L^{1/6}}{\left[1+\left\{0.492/P_r\right]^{9/16}\right]^{8/27}}\right\}^2$$

So, under these situations the turbulence in natural convection is defined when Rayleigh number at the transition which is Grashof number at the transition times Prandtl number is about 10^9. So, this is the limit for Rayleigh number for the case of turbulent flow. The most common examples of natural convection is when you have a vertical plate and you have natural convection taking place from its sides, and the correlation for finding out what would be the heat transfer coefficient under these conditions are given by Churchill and Chu where the Nusselt number relation is provided as:

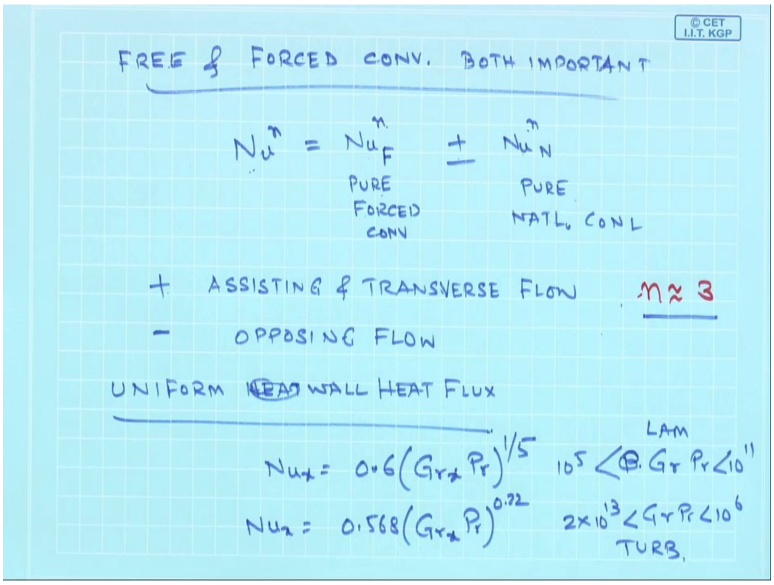
$$\overline{Nu}_L = \left\{0.852 + \frac{0.387 Ra_L^{\frac{1}{6}}}{\left(1+\left[\frac{0.492}{\Pr}\right]^{\frac{9}{16}}\right)^{\frac{8}{27}}}\right\}^2$$

. So, this is an impossible correlation to remember or to do anything with this. So, what the researchers have done over the years is they have huge number of experimental results under various conditions, and they start with some logic that Nusselt number has to be a function of Grashof number and Prandtl number or you can express it as a function of Rayleigh number and Prandtl number, and the dependence of the Nusselt number on the Rayleigh number or the Prandtl number these are obtained by fitting the experimental results with a suitable mathematical function, giving rise to these empirical expressions.

So, there are several such relations available in the text. I am giving you some of the examples to make sure you use the right correlation to find out the value of the heat transfer coefficient. All these correlations are valid for certain ranges of Rayleigh number. For example, turbulent flow and laminar flow the cross over takes place for Rayleigh number of about 10^9 and for certain ranges of Prandtl number. So, use those relations, but use them judiciously for the situation that you have at hand in order to obtain the value of h. But, the important thing, one of the reassuring things, is that no matter which correlation you use, the value of h is insensitive to some extent to the correlation that you have used. So, the value of heat transfer coefficient in natural convection is very small. For the case of air, it is going to be between 1 to 10 Watt per meter square per Kelvin. Since the value of heat transfer coefficient is small and it does not vary too much based on which correlation you use, your results are going to be approximately correct. But always consult your text or your hand book and choose the right relation for such cases.

So, the relations that I have shown you so far are valid only when natural conviction is predominant. But there would be situations in which both the natural convection and forced convection would be present. So, what would be the expression for such situations? I know separately how Nusselt number depends on Reynolds and Prandtl, for the case of forced convection and how Nusselt number depends on Rayleigh number and Prandtl number for the case of free convection. But, if both are significant, how do I get the overall Nusselt number for such a situation? So, that is what I am going to write next the case where both are important.

(Refer Slide Time: 30:32)



FREE & FORCED CONV. BOTH IMPORTANT

$Nu^n = Nu_F^n \pm Nu_N^n$

PURE FORCED CONV

PURE NATL. CONV

+ ASSISTING & TRANSVERSE FLOW

− OPPOSING FLOW

$n \approx 3$

UNIFORM ~~HEAT~~ WALL HEAT FLUX

$Nu_x = 0.6(Gr_x Pr)^{1/5}$ $10^5 < Gr\, Pr < 10^{11}$ LAM

$Nu_x = 0.568(Gr_x Pr)^{0.72}$ $2\times10^{13} < Gr\, Pr < 10^{6}$ TURB.

That means, you have both, free and force convection, and both are important. Then the effective Nusselt number is Nusselt number based on forced convection plus or minus Nusselt number based on pure natural convection. So, this plus is for the assisting and transverse flow whereas minus is for opposing flow. So, the Nusselt number for when both are present is simply an algebraic sum to the power some coefficient n where this n is generally taken to be equal to 3, and this is only a first approximation to find out what is the combined heat transfer coefficient when both are present. So, if we have both acting in the same direction, then it is going to be plus. If you have natural convection and force convection in the opposite direction, then you are going to have a negative sign.

The case when there is a constant surface heat flux is a very common situation which one would encounter in the case of natural convection where the Nusselt number is denoted by a relationship: $Nu_x = 0.6\left(Gr_x \Pr\right)^{1/5}$ and this is valid for $10^5 < Gr_x \Pr < 10^{11}$, while the flow is laminar. For the case of turbulent flow, the relationship is going to be: $Nu_x = 0.568\left(Gr_x \Pr\right)^{0.72}$, and this relation is valid for $2\times10^{13} < Gr_x \Pr < 10^{16}$. My idea is not to give you too many correlations because these correlations are availability in you text and you can look it up at any point of time. What I would like you to take away from this class is that, I need you to understand why natural convection is taking place. And since it is taking place, the pressure gradient force flow is provided by a difference in density caused by a difference in temperature. So, the body force term will have to be modified by considering the difference in density. The difference in density is related to difference in temperature. So, it is more logical to bring in a thermodynamic quantity beta which is the coefficient of volumetric expansion and write the equation of motion in a clear fashion which will highlight the role of temperature difference in natural convective flow along a flat plate.

So, this additional contribution from the buoyancy difference induced flow is clubbed in terms of a new dimensionless number which is known as Grashof number. So, if it is purely natural convection, my Nusselt number would be a function of Grashof number and Prandtl number. If it is a forced convection only then Nusselt number is a function of Reynolds number and Prandtl number. If both are present, then the effective Nusselt number is expressed as an algebraic sum of Nusselt number due to forced convection and Nusselt number due to free convection with an exponent n present in the relation. So, the values of n are generally taken to be equal to 3 and there are a multitude of relations which are available for Nusselt number under various conditions. So, the important thing to know here is that the value of the natural convective heat transfer coefficient is quite small as compared to its counterparts in forced convection. So, if we have a system in which forced convection is taking place inside a tube and free convection is taking place outside of the tube. The rate determining step, which is the one that offers maximum resistance to heat transfer, is going to be provided by the natural convection. The natural convective heat transfer coefficient or the inverse of that, because inverse of heat transfer coefficient is a resistance to heat transfer. Therefore, an idea of the natural convective heat transfer coefficient is very important in many of the calculations that you would encounter while designing the heat exchange equipment. So, identifying the flow pattern, identifying the role of natural convection in that process, and choosing the right correlation for that specific situation are all very important. The first two I have covered extensively in this class but choosing the relation or which relation is to be used and what would be the form of that equation, are all available in the text. So, I would encourage you to look at the text and see for what condition which correlation is to be used.

Heat Transfer
Prof. Sunando Dasgupta
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture – 48
Radiation - Fundamental Concepts

We will start Radiation for in this class and follow it towards the end of this course which would complete our study on Heat Transfer. So, far we have dealt with two modes of heat transfer, conduction and convection both of which require the presence of a medium. However, radiation is the one as you are all aware of does not require the presence of a medium. So, radiation plays a major role in many of the process industries and in significantly varied applications as we know of. So, we would concentrate on radiation, the radiator the radiation flux, the intensity and flux, the radiative properties.

How do we characterize the subject based on its radiative properties? What is an ideal surface as far as radiation is concerned? We are talking about black body over here. And then there is any surface resistance to radiation that by which you can express, since, radiation depends on temperature and on the nature of the surface. So, we will see what is Stefan Boltzmann law that; obviously, you know about. But, is there any surface resistance to radiation since all these surfaces and non ideal, they are not black bodies. So, can some resistance be ascribed for exchange of radiative heat transfer between the surface and its surroundings.

So, what is the surface potential? What is the surface resistance? What is the potential just outside of the surface when we consider the surface resistance? So, the body as a whole if I consider it as a black body we will have some sort of radiative potential. But, since its surface is not acting like a black body. So, there will be some resistance. So, just outside the surface we can think of another potential with two potentials connected by resistance as in a circuit, electrical circuit where the resistance in here is going to be surface resistance to radiation. So, those concepts we would also discuss in this part of the course.

Also two bodies can exchange radiation with in between them, that they can also exchange radiation with itself and they can they can also exchange radiative heat transfer with the surroundings. So, if the palm of my hand, if the palms of my hand are two surfaces are two different temperatures. And if I press them like this, there would be radiative heat exchange in between these two surfaces. Not only that they are also going to exchange heat with the walls of this room through the space in between this. So, if I bring them close to each other, the amount of radiative heat which they are going to exchange with the surrounding will keep on decreasing.

So, in ideally, if they are touching each other then all the heat which is released by one is going to be intercepted by the other. But, if I separate them out together then it is going to be, then the heat that gets transferred from one the fraction of the heat which gets transferred from one reaching two that fraction will decrease. So, the how much of surface 1 is visible from surface 2 would definitely depend on what is the distance between the two or if we talk in mathematical terms what is the solid angle subtended by that would by this by surface 2 onto 1. So, we will see the concept of solid angle as well in this course.

And then let us say we have 3 surfaces which form an enclosure. So, you can think of it as a triangle, where the 3 sides are exchanging radiative heat with one another ok. So, part of the heat released by 2 is going to go to 1, part going to 3 and maybe it may so, happen that part of the heat released by 1 is going to be absorbed by 1 itself. So, if it is a curved surface like this then the part of the energy which is emitted by 1 is going to incident on 1 itself right. So, that is possible, but if it is a flat surface then none of the radiative energy which leaves 1 is going to come back to 1.

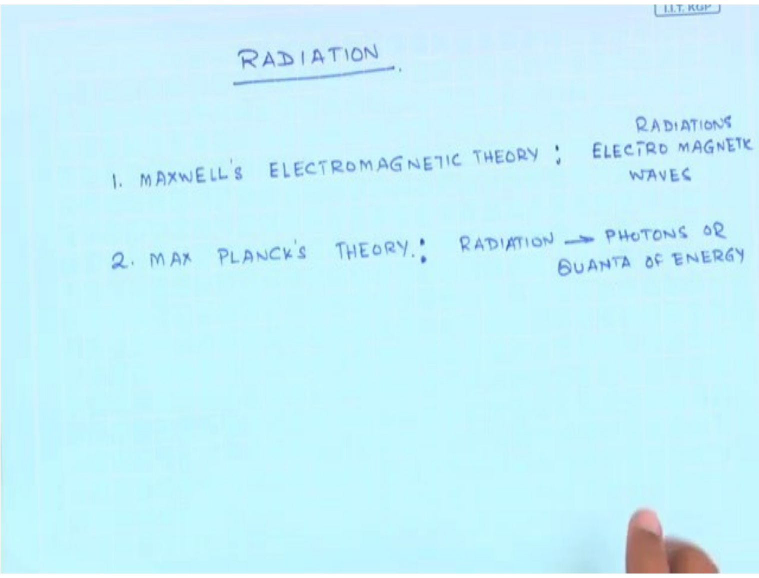
So, the nature of the surface, the shape of the surface will also play a role; let us say we have 3 flat surfaces which are forming a triangle. And these 3 sides are at 3 different temperatures and there is going to be radiative exchange of heat in between them ok. What is going to be the net heat to be supplied or extracted to each of these triangles, each of these sides of the triangle so, as to maintain thermal equilibrium? These informations will be extremely useful for the efficient designs of furnaces, because as you know furnaces operate at a very high temperature.

So, how much of heat you need to supply to the furnace to maintain the temperature of one of the surfaces at a desired level; you need this kind of calculations. So, as you can see we are always bringing in electrical analogy for radiative calculations specially when we talk about the potential, the resistance and the heat flow is something which is similar to that of current. So, there has to be a network method, see if you remember your electrical technology this star delta connections the problems on star and delta connections which we have done in electrical technology, we are going to do similar calculations for the case of radiation as well which would give, which would allow us to effectively design a furnace.

So, that is more or less what we would like to cover in this part of the course. But, let us start with radiation first and radiation is the energy emitted by an object which does not require the presence of a medium and all surfaces above absolute 0 would emit radiation. They will also receive radiation so, there is going to be a process of emission and absorption that are going on that will be going on.

The concept of radiation can be explained by 2 theories, the first one is Maxwell's theory and the second one is Planck's theory. So, Maxwell's theory of electromagnetism can be used to express some of the radiation phenomena where Max Planck's theory is also applicable for the case of radiative heat exchange. So, apart from the properties of the surface we should also be aware from the very class first class itself that the radiation can be explained either in Maxwell either using Maxwell's theory of electromagnetic, Maxwell's electromagnetic theory where the radiation is treated as electromagnetic waves.

(Refer Slide Time: 08:05)



RADIATION.

1. MAXWELL'S ELECTROMAGNETIC THEORY : RADIATIONS ELECTRO MAGNETIC WAVES

2. MAX PLANCK'S THEORY : RADIATION → PHOTONS OR QUANTA OF ENERGY

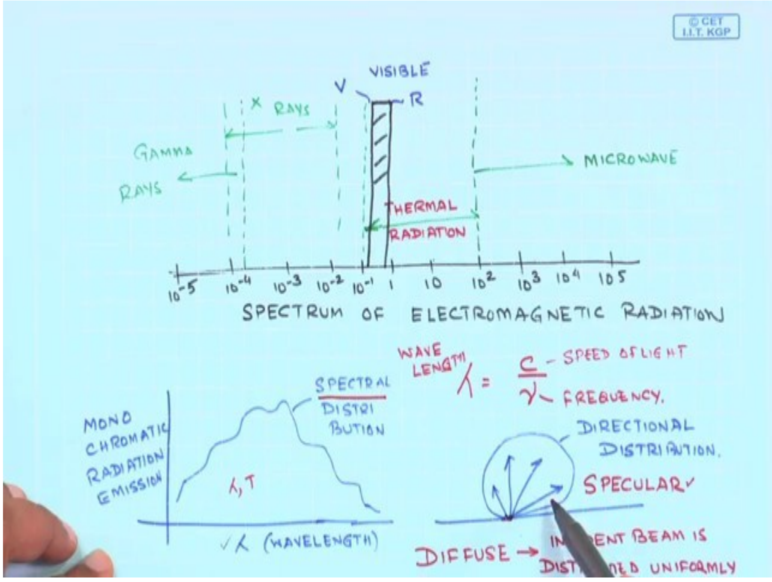
So, radiations are treated as electromagnetic waves in Maxwell's electromagnetic theory wherein max Planck's theory the radiation is treated as photons or in other words they are Quantas, quanta of energy. So, both we will find its use, both concepts have been utilised. So, the results from the electromagnetic theory are used to predict the radiative properties of materials, while the results from Planck's concept has been used to predict the magnitude of radiation energy emitted by a body at a given radiation.

Now, radiations can happen over a large wavelength range and view on one side we have the gamma rays and one on the other side we have radio waves. So, whenever an object is emitting radiation the chances are that it is going to emit radiation over all the wavelengths. So, radiation is spectral in nature by spectral I mean that there is going to be a wavelength dependence of radiation and not all the energy coming out of a surface is going to be equally spaced over all wavelengths. So, there will be certain wavelengths, there will be a wavelength range in which most of the energy would be concentrated and it may not be presented other wavelengths. So, the spectral nature of radiation is extremely important and we will have to take that into account.

So, when you think of this spectrum of radiation and the way it is divided into different different sections. So, there is a specific wavelength range where we can see that is the visible wavelength range. There is a specific wavelength range we in which most of the radiative heat transfer is taking place. And you have an infrared range and you have the gamma rays range where you have energy associated with radiation. But, the thermal energy is concentrated somewhere in the middle and that is going to be our zone of interest for most of the topics that we would cover over here.

So, let me draw what kind of waves, what kind of radiation we would get and as a function of the wavelength and see whether we can identify and demarcate the region in which the radiation is going to be important. So, if I draw that profile over here.

(Refer Slide Time: 11:41)



What it looks like then is. So, this is the spectrum of electromagnetic radiation. So, any wavelength, any radiation where the corresponding wavelength, where the associated wavelength is greater than 10^2, these are in the microwave region. Anything which is smaller than 10^{-4} these are gamma rays. In between 10^{-4} and roughly somewhere slightly more than 10^{-2} these are the x rays. So, between 10^{-1} to about 10^2, this is the region which is of most importance to this part of the course which is thermal radiation; so, between 10^{-1} to about 10^2.

And this region this one is the visible range where you have the red on one side and you have the violet on the other side. So, this is more or less the visible spectra the more or less the spectrum that you would get in radiation, but between 0.1 and 100 this is the range in which most of the thermal radiation is going to be concentrated. And this would be the region which we would study in this part of the course. So, as well as I was telling you the spectral nature and the so called specular nature. So, this is monochromatic light, monochromatic radiation emission and if I plot it as a function of λ which is wavelength. There is going to be a variation like this some arbitrary variation. And since, it depends on wavelength this is called spectral distribution.

What it essentially tells is that you are getting a continuous non uniform distribution of monochromatic components, by monochromatic I mean single wavelength components and the magnitude of the radiation at any wavelength and the spectral distribution vary with the nature and temperature of the surface. So, this kind of distributions what you are getting out of the surface, out of the object, would depend on whatever be the temperature of the object and whatever be the properties of the object.

So, the monochromatic distribution of radiative emission from a surface is a continuous function of wavelength, but the value of the emission is going to be different at different wavelengths. So, that is what happens, that is what would happen in most of the real surfaces. Whereas in some cases your and there is no directional distribution on top of it. But, if you look at this surface then from a point it is this one will have a directional distribution which is something different from over here.

So, here the distribution depends only on wavelength, here the distribution depends on the direction as well ok. So, this is the difference between these two surfaces. So, this is a spectral distribution and the spectral is used to refer to the nature of the dependence where it depends on the wavelength. And the thick the wave nature of the thermal radiation tells you that this λ which is the wavelength is going to be (c)/ (frequency of radiation). So, this is the speed of light that is the standard definition speed of light.

The speed of propagation in the medium and λ is the wavelength and this is the frequency. So, if the medium through which the propagation takes place is vacuum, then that c is going to be speed of light in vacuum. So, the dependence between the wavelength and frequency is simply an inverse relationship with a constant which is the speed of light in vacuum when we are talking about the vacuum. So, now, let us talk about the radiative properties, what are the radiative properties that we have? When a light is incident on a surface part of it is going to get reflected, part of it is going to be absorbed and part of it is going to get transmitted.

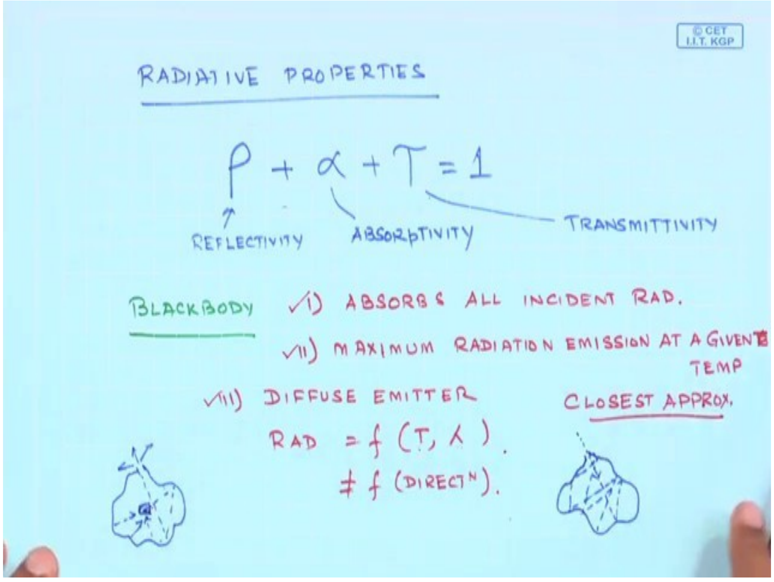
So, the fraction of the energy which gets reflected as compared to what is the overall energy incident on it is known as the property is the reflectivity. So, the reflectivity is defined as the fraction the numerator is going to be the fraction which is reflected fraction of energy which is reflected and the denominator is the total energy incident on it. So, the other option, other possibility is that part of the energy which does not get reflected and enters through the surface is going to get absorbed. So, the absorptivity, the factor absorptivity is defined as the amount of energy which is absorbed in the material divided by the amount of energy which is incident on it.

Now, let us say that part is reflected; part gets into the system and then goes out of the system or out of the material after part of it is going to be absorbed. So, what can happen on incident energy on a surface is one it may get reflected, one it may get absorbed and the third is it may get transmitted through the surface. So, the three properties which are defined to denote these natures of the surface are reflectivity, absorptivity and transmissivity. As the name suggests the denominator in all cases would be the amount of energy incident on the surface.

So, in the case of reflectivity it is a fraction which gets reflected. Absorptivity is the fraction which gets absorbed and transmissivity is the fraction which gets transmitted. So, the reflectivity, the absorptivity and transmissivity, transmissivity all are fractions having a value from 0 to 1. And since the denominator in all cases is the same, the sum of alpha plus beta plus gamma tau is going to be equal to 1. So, if I define the properties,

$\rho + \alpha + \tau = 1$

(Refer Slide Time: 22:15)



© CET I.I.T. KGP

RADIATIVE PROPERTIES

$$\rho + \alpha + \tau = 1$$

REFLECTIVITY

ABSORPTIVITY

TRANSMITTIVITY

BLACKBODY

✓i) ABSORBS ALL INCIDENT RAD.

✓ii) MAXIMUM RADIATION EMISSION AT A GIVEN TEMP

✓iii) DIFFUSE EMITTER

RAD $= f(T, \lambda)$.

$\neq f$ (DIRECT^N).

CLOSEST APPROX.

So, this is the reflectivity, this is the absorptivity and this is the transmissivity; so, fraction of fraction which gets reflected, fraction which gets absorbed and fraction which gets transmitted. So, that is why sum of these 3 would be equal to 1. Now, if I go back one slide and show you this one here this one is this kind of surfaces where the emission depends on the direction they are known as specular. So, specular is a surface where there is a directional dependence of radiation from a surface.

So, wherever there is a directional dependence or directional distribution of energy out of a surface it is known as specular. And when there is no such dependence it is called a diffuse. A diffuse is a surface where the incident beam incident beam of energy is distributed uniformly in all directions. So, apart from these properties of radiation, this property of the surface is also important. Specular when there is a directional distribution to the energy emitted by a surface. The second type of surface is diffuse where the incident beam is distributed uniformly in all directions.

So, we will see what are the examples and the special features and the simplifying situations where assuming a surface specular or assuming a surface as diffuse we will provide. The other concept that I would like to introduce before I close this class is some sort of an ideal. Whenever we talk about real surfaces the properties the absorptivity, reflectivity and the transmissivity there must be something which can be taken as an ideal surface ok. So, we are going to compare the performance or the characteristics of a real surface against that of the ideal.

So, such ideal surface is in radiative heat transfer is assumed is termed as the black body. So, what is a black body? A black body is something it is an idealized concept; a black body is something which absorbs everything that is incident on it. So, therefore, the absorptivity of a black body is equal to 1. On the other hand the second characteristics of a black body is that at a given temperature no other object emits more energy than that of a blackbody. So, blackbody emits the emissive power of the black body is maximum compared to all other surfaces. So, black body absorbs anything and it emits the maximum amount of energy.

The concept of black body is extremely important in radiative heat transfer. Because, it lets you set a standard surface against which the performance of all other surfaces are to be evaluated. So, let us note down the special nature characteristics of a black body. It is an idealized surface, idealized concept and the first one is absorbs everything as I said. The second one is maximum radiation at a given temperature that is the second one and the third one that black body is a diffuse emitter. As I said the radiation from a black body is a function radiation is a function of temperature and wavelength. But it is not a function of direction.

If you again compare this, what we have done in this case the radiation takes place at all wavelengths and the amount of radiation is going to be a function of temperature. So, if I increase the temperature I am going to get a second curve which would show that it is going to provide higher amount of energy so, emission. So, the radiation emission is definitely a function of λ it is also a function of temperature. So, common sense tells us that more the temperature higher is going to be the emission. Apart from that there could be directional dependence of radiation what we call as specular, where there is the from the point there is going to be different in different directions it would emit different amounts of energy.

So, ideally the radiation is a function of temperature, it is a function of wavelength and it is function of direction. But, in for case of a black body if the radiation emission is not a function of direction, it is a function only of temperature and that of λ. So, when a surface is not a function of. When the emission from a surface is not a function of the direction it is called diffuse emitter. So, in order to be a blackbody these 3 conditions are to be satisfied that it should observe all incident radiation coming at all possible coming from surfaces at all possible temperatures and all possible wavelengths.

And at a given temperature it is going to emit the maximum amount of radiation that you can think of and it is going to be a diffuse emitter. So, examples of there are some surfaces which closely mimic the behaviour of a black body for example, the closest approximation of a blackbody. The closest approximation of a black body can be obtained when you think of a small cavity. So, this is a small cavity which you can think of it with a very small opening over here. So, any ray which comes inside is going to be reflected multiple times and the chances of multiple time it is going to reflected and the chances of its going back outside through the narrow pore is extremely small.

So, what it tells us is it gives us the situation in which any energy which comes inside is going to be absorbed which it is going to be absorbed multiple times inside the system and the fraction of energy which may go out is going to be extremely small. So, it obeys or it confirms to the first approximation of a black body; that means, anything which is incident on it is going to be absorbed over here. Now, let us think of the surface once again the chances of the emission that you are going to get out of this is same in all direction. So, if something leaves through this pore, it can be in any possible direction. So, therefore, this is one of the closest approximations that you can get of a black body and let us assume that you have placed an object over here.

In the one which enters is going to reflect multiple times and come to it from all possible directions right. So, energy is going to be incident on any object placed inside the cavity. So, therefore, it is going to be a diffuse irradiation of interior surfaces. Since, it is coming all surfaces all points on the surface. So, the body placed inside the cavity will experience evaporation from all possible directions. So, therefore, the black body is going to be a diffuse emitter, since it is going to irradiate at the object inside the cavity equally with equal probability from all directions.

So, normally a cavity is the closest approximation of a black body that one can get. So, the properties of blackbody, the radiative properties of any real material, how they differ from that of a black body and what is the closest approximation of a black body we have discussed that. But, what is left to be said is, what is the black body radiation intensity. Is there a formula, is there some way by which we can find out what is the spectral and once again spectral means wavelength dependance. What is the spectral radiation intensity of a black body? As we understand it is going to be a function of temperature, it is going to be a function of wavelength. There it is going to be some other constants which are going to be involved in that expression.

So, in the next class will see: what is the spectral radiative intensity of a black body and how this can be connected with the radiation flux from a point source. If I put a point source of radiation on a given surface, how much of total radiation I am going to get out of the surface.

So, that is what we will cover in the next class.

Heat Transfer
Prof. Sunando Dasgupta
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 49
Spectral Blackbody Radiation Intesity and Emissive Power

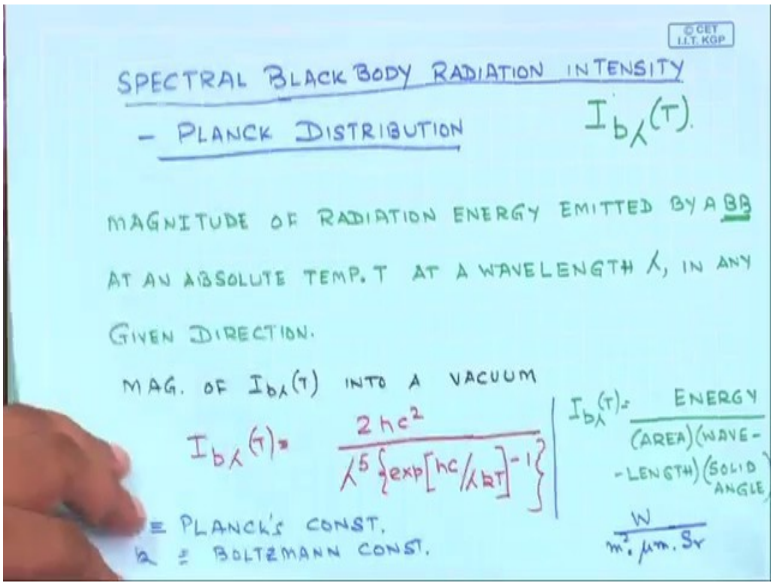
We will discuss more about radiation in this class. In the last class we just introduced you to radiation, the requirement of not having any medium; that means, a radiation can take place without any medium being present. The radiative properties of a substrate in terms of reflectivity, absorptivity and transmittivity and the concept of an ideal body in terms of radiation, which is termed as a black body which can emit; whose emission at a given temperature is the maximum, it is going to absorb everything that falls into it. And, it is a diffuse emitter; that means, the emission coming out of a black body does not have any directional dependence.

We have also introduced the concept spectral; that means, anything which depends on the wavelength at which we are considering at which the emission is taking place. So, all these quantities, all these properties namely the reflectivity, the transmittivity and the absorptivity are spectral in nature; that means, the absorption of a radiation is going to be different at different values of the wavelength. So, the spectral nature of radiation concept of a black body and that the radiation can be treated in 2 ways, the one is based on the wave nature and the second is based on the quantum concept these were introduced in the last class. In this class we are going to know more about two fundamental properties of a black body.

That means, first is going to be: what is the black body radiation intensity, when a black body is at a given temperature. So, how much of energy per unit area per unit wavelength and per unit solid angle we will introduced the concept later on that this black body is emitting. So, that is the spectral black body radiation intensity.

And secondly, we will see what is going to be the total emissive power of a black body at a given temperature; that means, if a black body having unit area is placed at the center of a hemisphere, how much would be the total energy emitted by this black body of unit area at a given length and at a given temperature. So, these are the two main concepts which would cover in today’s class.

(Refer Slide Time: 02:57)



SPECTRAL BLACK BODY RADIATION INTENSITY

– PLANCK DISTRIBUTION $I_{b\lambda}(T)$.

MAGNITUDE OF RADIATION ENERGY EMITTED BY A BB AT AN ABSOLUTE TEMP. T AT A WAVELENGTH λ, IN ANY GIVEN DIRECTION.

MAG. OF $I_{b\lambda}(T)$ INTO A VACUUM

$$I_{b\lambda}(T) = \frac{2hc^2}{\lambda^5\left\{\exp\left[hc/\lambda kT\right]-1\right\}}$$

$$I_{b\lambda}(T) = \frac{\text{ENERGY}}{(\text{AREA})(\text{WAVE-LENGTH})(\text{SOLID ANGLE})}$$

$$\frac{W}{m^2.\mu m.Sr}$$

= PLANCK'S CONST.

k = BOLTZMANN CONST.

So, let us first start with black body radiation intensity in which is going to so, we denote that as I the intensity, b stands for the black body, lambda denotes the wavelength dependence and of course, it is going to be different and different temperature. So, the temperature has to be specified while we are discussing about the black body radiation intensity. And in this case, we are going to rely on the Planck's distribution. So, what is black body radiation intensity? It is defined as the magnitude of radiation energy which is emitted by a black body at an absolute temperature T at any wavelength lambda in any given direction.

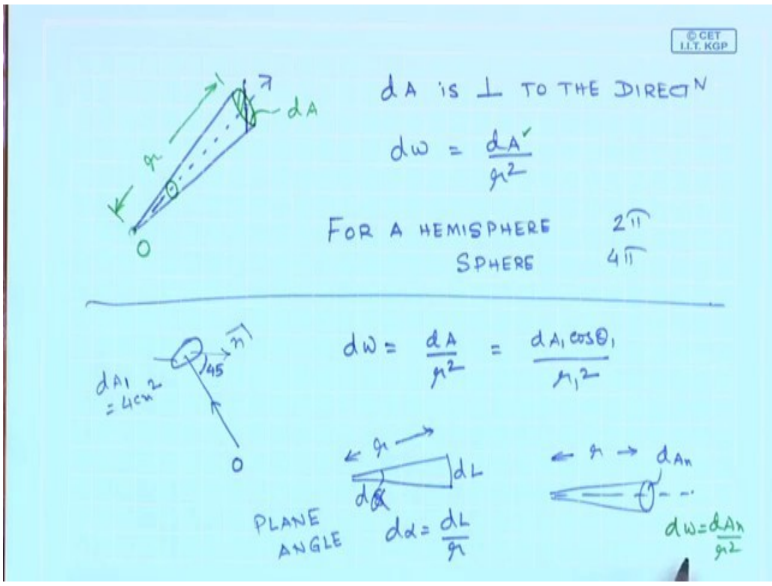
So, it gives you the idea the quantum of radiation energy which is emitted at in any given direction and at a specific wavelength. So, the radiation energy of course, would therefore, depend on the temperature, it is going to depend on the lambda, the wavelength and it will have a directional property.

So, using Planck's distribution the magnitude of the black body radiation intensity can be expressed for the case of vacuum. So, the magnitude of the black body radiation intensity, spectral black body radiation intensity at a given temperature into vacuum is expressed in this form. Where h is the well-known Planck's constant. So, h is the Planck's constant, the value of which is available in any text book I am not writing it over here; c is the, if c is the velocity of light and the k the k that you see over here is the Boltzmann constant. So, that is also the value of which is also available in your text.

So, if you if you think about what would be the units of the black body radiation intensity from the definition we understand that it is going to be energy per unit area per unit wavelength per unit solid angle because, we are talking about in any given directions. So, I will tell you what this solid angle is all about, but the definition got the units of the black body radiation intensity therefore, can be expressed in terms of energy which is Watts, area, wavelength is customarily expressed in terms of micron. So, that is why the micron is there in the denominator and the solid angle is the unit of solid angle is steradian. So, steradian comes into the denominator.

So, the units of the black body radiation intensity I b lambda T, the spectral blackbody radiation intensity is Watt per meter square per unit per micron per steradian. So, what is a solid angle let us since we are going to use the concept of solid angle.

(Refer Slide Time: 06:15)



You are probably aware of this solid angle, but I will still do this one more time. So, let us see this is the over here therefore, it is sort of a cone which is my area of interest let us say the area is dA. The distance from the center from this point which I denote it as O let it be r. So, this is the solid angle subtended by the area dA at the center and this dA is normal perpendicular to the direction.

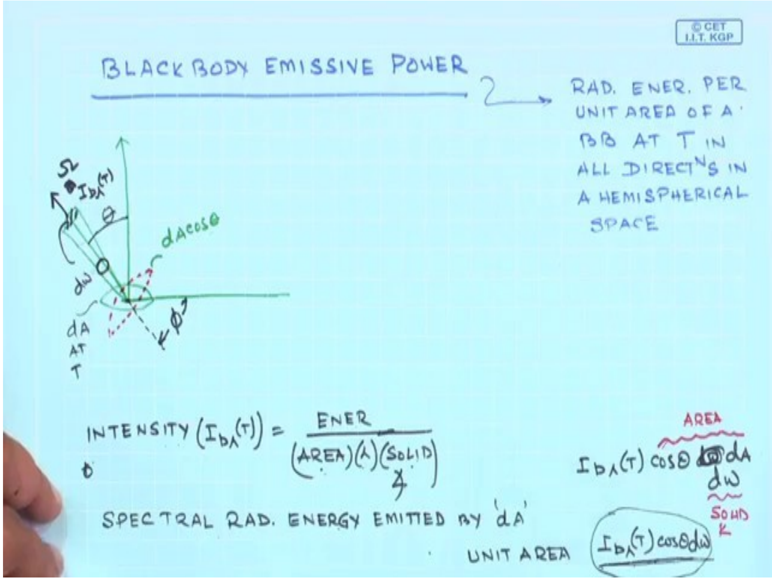
So, therefore, this dA perpendicular to the dotted line and the solid angle is defined as the area times r square. Had this area been at an angle with the line which comes out of this origin the specifying the direction then the projection of this area perpendicular to the direction has to be provided. That means, dA has to be substituted by dA cos theta if this is the area through which the radiation is passing through. So, therefore, the projection of this area on to this side has to be substituted for dA. So, dA is therefore, perpendicular to the direction and this has to be kept in mind.

So, if you extend this so, for a hemisphere this if the from the center, from its center the solid angle is simply going to be equal to twice pi. And when you consider as full sphere this is simply going to be equal to 4 pi. So, these are obvious; that means, what is going to be, what is going to be this value of the solid angle for the case of a hemisphere and for the case of a sphere. So, as before if let us say as I was telling you if this is the origin and I have some area which is not perpendicular to which is not perpendicular to this and therefore, let us say the area vector makes an angle of 45 degree with this direction, with this direction.

And let us say if this d area, the area is about 4-centimeter square. So, this is the area vector which is always perpendicular to this area. So, therefore, using the definition d omega the solid angle as dA by r square and this dA is simply going to be the area which is let us dA 1. So, this must be equal to dA 1 the projection of dA 1 in this direction. So, this is going to be dA 1 cos theta 1 divided by r 1 square. So, this is how the solid angle is evaluated.

So, if it is just part of a circle and if this is dL at a distance of r then the plane angle this alpha let us call it as d alpha. So, d alpha is simply going to be dL by by r, that is this standard relation when we were talking about a plane angle. And, when we are talking about a solid angle like this, if this is the distance is r and this area is dA n then in that case the d omega is simply going to be dA n by r square, where A n is the area normal to this direction. So, that is what we call is that plane angle and what is known as the solid angle. So, next thing is important because, next going to give us some more insights into the whole process.

(Refer Slide Time: 10:55)



BLACK BODY EMISSIVE POWER ? → RAD. ENER. PER UNIT AREA OF A BB AT T IN ALL DIRECTNS IN A HEMISPHERICAL SPACE

INTENSITY $(I_{b\lambda}(T)) = \frac{\text{ENER}}{(\text{AREA})(\lambda)(\text{SOLID} \angle)}$

SPECTRAL RAD. ENERGY EMITTED BY 'dA'

$I_{b\lambda}(T)\cos\theta\,\underbrace{dA}_{}\,\underbrace{d\omega}_{}$ AREA SOLID ∠

UNIT AREA $I_{b\lambda}(T)\cos\theta d\omega$

And it is a very important quantity which is known as the black body emissive power. So, what is a black body emissive power? The expression for which we need to find out so, what is black body emissive power? It is the radiation energy per unit area of a black body at T at an absolute temperature T in all directions in all directions in a hemispherical space. So, this is what is known as the black body emissive power and we need to find out what would be it is expression based on our knowledge of blackbody radiation intensity.

So, whereas, black body radiation intensity is in a given direction the black body emissive power is in all directions in a hemispherical space. So, the difference between these 2 must be kept in mind in one case the radiation intensity specifies a direction. So, it is the direction which is specified for the case of radiation intensity whereas, for the case of emissive power it is assumed that the black body is placed at the center of a hemisphere. And, we are trying to see what would be the total energy emitted by this black body of unit area which is placed at the center of the hemisphere in all possible directions. So, we need to find out we understand that this I b lambda T which we have defined previously it has a directional dependence. So, the solid angle subtended by the area at a distance from the center at a distance from the center is used to obtain what is the black body radiation intensity. Now, what we have to do is we have to make this area which is subtending a solid angle over here travel in such a way that it defines a hemisphere.

So, if we can integrate the area in such a way that the entire hemispherical dome over this unit surface area can be specified then the total energy which passes in all directions in the hemispherical space at a given temperature will be known as the black body emissive power. And, that is what we are going to evaluate we are going to derive in this in this part of the class.

So, from the intensity now we are going towards emissive power. In one case the area is fixed it is dA 1 and we are trying to see what is the solid angle in the other case this area is essentially the area of the hemisphere. So; obviously, the solid angle will change depending on where this unit area is placed. So, this has to be taken into account while evaluating this. So, let us draw the draw this first and see what we get out of this. The first one let us say we have this is the area vector, this is the area, we would like to find out what is going to be the intensity in this specific direction.

So, this is the preferred direction let us call it is omega and in this direction the intensity is I b lambda T. The area over here is this is the area we are talking about and initially this is the area dA at a temperature T. But, as you can see it is not perpendicular it is not perpendicular to the direction of propagation of to the direction in which I would like to find out I b lambda T.

So, what I do is I try to see: what is the azimuth angle of this area over here and this being the angle theta. So, what I do is I will draw the projection of the green one dA and therefore, this area is simply going to be dA cos theta. So, my dA cos theta is now perpendicular to the direction in which I would like to find out what is I b lambda T. Once again, the I b lambda T is the black body radiation intensity. In fact, spectral black body radiation intensity at a given temperature in this at a specific direction which is a this direction.

The object from where this intensity is coming is dA. So, I am going to take a projection of this make the area of perpendicular to the direction. So, therefore, this area is to going to be dA cos theta. So, if I write the intensity in this case which is I b lambda T should equal to energy per unit area per unit wavelength and per unit solid angle. So, this is my definition of the energy.

So, the spectral radiation energy let us the spectral radiation energy emitted by emitted by dA by dA the surface element dA in which which through an elemental solid angle d omega. So, this angle solid angle is d omega. So, this d omega I would like to find out how much of how much of spectral radiation energy emitted by dA passes through the solid angle d omega. So, this is the energy which is contained within this tube that I would like to find out ok.

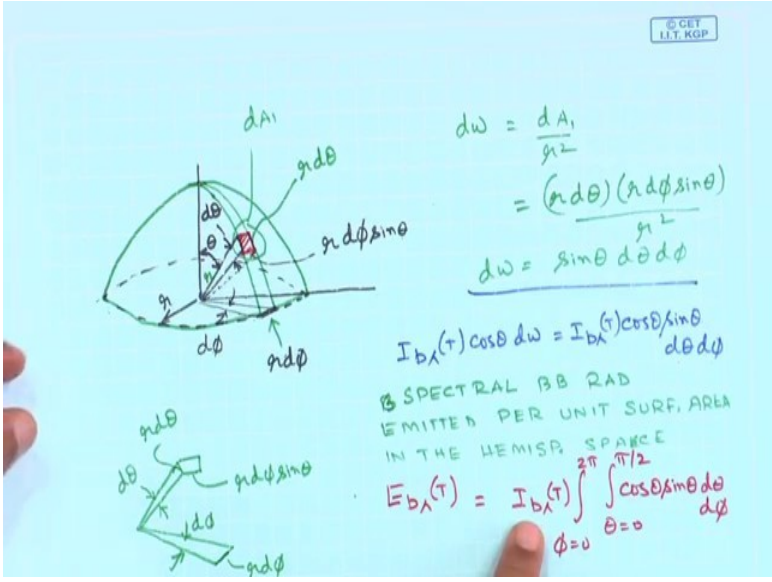
So, this must be equal to I b lambda T then cos theta d omega times dA d omega. So, this energy; obviously, would be in order to obtain the energy I need to multiply the intensity with area. So, this is going to be my area, the solid angle is this. So, this is the solid angle, this is this is what is going to be the energy the spectral the spectral energy. So, if I looked like to find out the energy by unit if I want to do it with in terms of unit area, it should simply be equal to I b lambda T cos theta d omega.

I will go through it once again; my intensity is defined as energy per unit area per unit wavelength per unit solid angle ok. So, the area is dA which makes an angle of theta with the area vector. This d this dA and I would like to find out how much of energy is going through this tube which forms a solid angle dA at this point. So, in order to do that the first thing is I need to make sure that this area is placed in a direction perpendicular to this.

So, which is going to be dA cos theta and this is this is this is the azimuth angle which we will discuss later on. So, the energy per unit wavelength or in other words the spectral energy would therefore, be the product of intensity times area which is perpendicular to the direction times the solid angle. So, that in then it should be the spectral radiation energy emitted by dA would be the intensity times area which is dA cos theta times solid angle which is d omega. If you like to find out what is the spectral radiation energy emitted by an unit area I simply divided by dA and this is the expression of the spectral radiation energy emitted by and unit area instead of dA an unit area.

So, this is the quantity which I am going to use, but in order to effectively use this quantity I need another factor ok. So, another figure so, the figure that I am going to use going to draw is this one.

(Refer Slide Time: 21:17)



$$d\omega = \frac{dA_1}{r^2} = \frac{(r\,d\theta)(r\,d\phi\sin\theta)}{r^2}$$

$$d\omega = \sin\theta\,d\theta\,d\phi$$

$$I_{b\lambda}(T)\cos\theta\,d\omega = I_{b\lambda}(T)\cos\theta\sin\theta\,d\theta\,d\phi$$

B SPECTRAL BB RAD EMITTED PER UNIT SURF. AREA IN THE HEMISP. SPACE

$$E_{b\lambda}(T) = I_{b\lambda}(T)\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2}\cos\theta\sin\theta\,d\theta\,d\phi$$

So, this is a hemisphere which I am drawing of base radius r and this is a slice of that which where the angle is d phi. And once I draw this it would be clear to clear to you. The angle over here is theta and this angle, this small angle is d theta ok. And this is d phi so, this one must be equal to r times d phi because this is a plane angle where the radius is r and the angle is d phi. So, the length of the chord must be equal to d phi. This when we go all the way up to 90 minus theta I get this line, if I go all the way to 90 I get a point. If I go up to this point this is simply going to be r d phi sin theta ok.

So, when sin theta becomes equal to 90 which is at the top then would this would be equal to 0. So, the length of the line will vanish, and it will become a point. When you think of this one, this is d theta and the length the this thing the radius is r. So, therefore, this one is simply going simply going to be equal to r times d theta.

So, if I project it slightly in a better way this is what you get as r d phi. And when you go all the way up to this point so, this is d phi, this is d theta. So, what you get like this the area this length is r d theta and this length has to be r d phi sin theta. So, my area which is defined by the angles phi and theta is r d theta sin theta is 1 one dimension, the other dimension is r times d theta.

So, therefore, from this figure my d omega the solid angle is simply going to be equal to dA 1 divided by r square where dA 1 is this area. So, this area is dA 1, the area which is denoted by the red crosses and this dA 1 is simply the product of these 2 length scales.

So, it is going to be r d theta is one length one side the other side is r d phi sin theta divided by r square. So, this d omega is simply sin theta d theta d phi that is going to be that is going to be the solid angle.

So, when we go back to this figure once again I have my spectral radiation energy emitted by unit area is I b lambda T cos theta times d omega. So, I need to put instead of d omega in this expression the d value of d omega that I have obtain. So, therefore, I b lambda T cos theta d omega would simply be equal to I b lambda at a constant temperature times cos theta sin theta d theta d phi. So, this is this is going to be the spectral radiation energy emitted by an unit surface area element through which substance a solid angle equal to phi over here now.

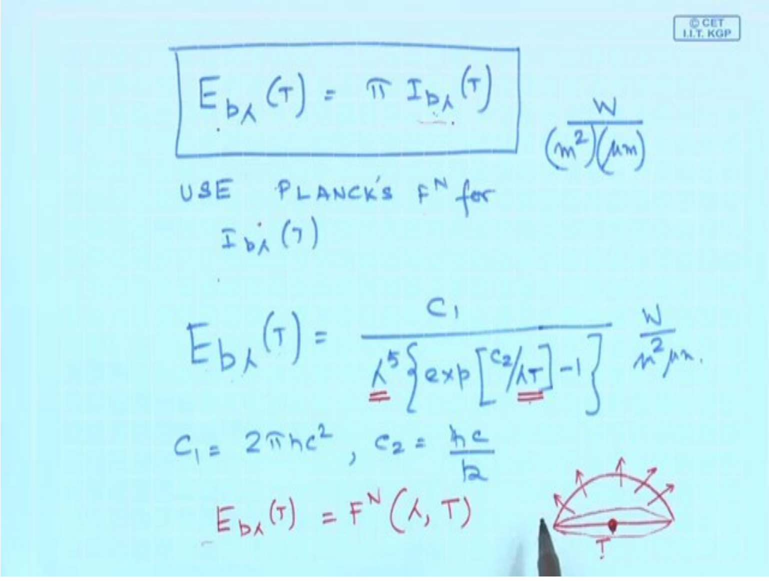
Now, I would like to make integrate this expression in such a way such that this area is going to represent the entire hemispherical area. So, if I can do the integration in such a way that this area is going to encompass the entire hemispherical area. So, I can see that in that integration my theta is going to vary from 0 to 90 degree, 0 to 90 degree and my phi is going to vary from 0 to 2 pi that is what the variation is going to be. Let us look at it to another way this is what I am trying to do.

So, in order to create the hemispherical space my phi is going to be from 0 to 2 pi whereas, my theta is going to be from 0 to phi. So, if I can let this area travel in terms of theta from 0 to 90 degree and in terms of phi from 0 to 2 pi then this area encompasses the entire hemispherical area available to exposed when the black body is placed at the center.

So, the black body the spectral skill depends on the wavelength the spectral blackbody radiation which is emitted per unit surface area in the hemispherical space in the hemispherical space. Hemispherical space which is denoted as E b lambda T this is what is the spectral black body radiation emitted per unit surface area into the hemispherical space would simply be I b lambda T which is outside of the integration sign. And phi would be from 0 to 2 pi and theta would be from 0 to pi by 2 and inside would be cos theta sin theta d theta d phi.

One more time since it is black body radiation intensity so, it is independent of the direction, since the black body radiation is diffuse. So, I b lambda T does not depend on the direction and therefore, it can be taken out of the integration sign. So, what is left is cos theta sin theta d theta d phi and as I have explained the theta the phi is going to vary from 0 to 2 pi and theta is going to vary from 0 to pi by 2. If I perform this integration I have an expression of E b lambda T in terms of I b lambda T. So, what that expression would be? Once you perform this one once you perform this integration it is there in your text.

(Refer Slide Time: 30:17)



$$\boxed{E_{b\lambda}(T) = \pi I_{b\lambda}(T)} \quad \frac{W}{(m^2)(\mu m)}$$

USE PLANCK's F^N for $I_{b\lambda}(T)$

$$E_{b\lambda}(T) = \frac{C_1}{\lambda^5 \left\{ \exp\left[C_2/\lambda T \right] - 1 \right\}} \quad \frac{W}{m^2 \mu m}.$$

$C_1 = 2\pi h c^2$, $C_2 = \frac{hc}{k}$

$E_{b\lambda}(T) = F^N(\lambda, T)$

I am not going to do it over here it is a very simple integration. This is the expression for body radiation the spectral blackbody emissive power. So, since it is so, it is energy per unit area per unit wavelength that is going to be it is unit energy per unit area per unit wavelength. So, the spectral black body radiation intensity is related to spectral emissive power of an unit based on an unit area. In a hemispherical space is denoted by specific relation.

So, when you use function for I b lambda T E b lambda T would be c 1. So, the emissive power the spectral emissive power of a black body of unit area at a given temperature is provided as a function of wavelength and as a function of temperature. So, this is the important part.

So, E b lambda T is a function of lambda and is a function of temperature. This specific expression will be utilized in the next class to show how the emissive power of a blackbody spectral emissive power of a black body depends on the wavelength and depends on the temperature.

So, what we have done in this class is we have placed a black body of unit area inside hemispherical dome. And, we have found out what is the total emissive power of this black body having unit area at a given temperature, what is the spectral power emissive power of black this black body into the hemispherical space.

So, this is what it looks like, in this hemispherical space how much of radiative energy a black body of unit area at a given temperature is providing. So, this is related to the intensity of radiation and we know the intensity of radiation through the use of Planck's function.

And then we can find out E b lambda T as a function of the wavelength and as a function of temperature. So, this functional relation we will explore a bit further and then you would see that it is going to give rise to certain relations that we know of; for example, the most common example or relation of radiation the Stefan Boltzmann law can be directly derived from this blackbody emissive power.

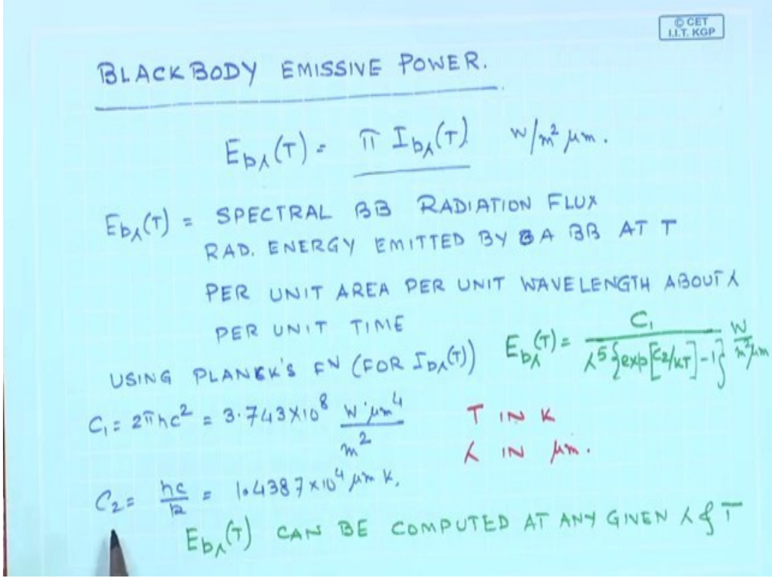
So, we will do that in the next class.

Heat Transfer
Prof. Sunando Dasgupta
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture – 50
Weins's Law, Stephen Boltzmann Law, Blackbody Radiation Function, Tutorial Problem

In the previous class, we spoke about the Black body Emissive Power and we wanted to see what is the relation between the Black Body Emissive Power commonly denoted by $E_{b\lambda}$ at a given temperature with the black body radiation intensity. And the relation which we have obtained considering a black body of unit area placed at the centre of a hemisphere and measuring what is the total energy emitted per unit time per unit area per unit wavelength by a black body into the hemispherical space. We have obtained the relation between the emissive power and the intensity of blackbody radiation.

(Refer Slide Time: 01:07)



BLACKBODY EMISSIVE POWER.

$E_{b\lambda}(T) = \underline{\pi I_{b\lambda}(T)}$ W/m² µm.

$E_{b\lambda}(T)$ = SPECTRAL BB RADIATION FLUX
RAD. ENERGY EMITTED BY ~~B~~ A BB AT T
PER UNIT AREA PER UNIT WAVELENGTH ABOUT λ
PER UNIT TIME

USING PLANCK'S FN (FOR $I_{b\lambda}(T)$) $E_{b\lambda}(T) = \frac{C_1}{\lambda^5\{\exp[C_2/\lambda T]-1\}}$ $\frac{W}{m^2\mu m}$

$C_1 = 2\pi hc^2 = 3.743\times10^8 \frac{W\,\mu m^4}{m^2}$ T IN K

λ IN µm.

$C_2 = \frac{hc}{k} = 1.4387\times10^4$ µm K.

$E_{b\lambda}(T)$ CAN BE COMPUTED AT ANY GIVEN λ & T

So, the relation that we have obtained is the following

$$E_{b\lambda}(T) = \pi I_{b\lambda}(T) \text{ W/m}^2\mu\text{m}$$

where $E_{b\lambda}(T)$ is the spectral black body radiation flux which is essentially the energy emitted by a black body at any given temperature T. We need to remember that this T, the temperature in radiation always does refer to the absolute temperature per unit area per unit wavelength about that wavelength λ per unit time.

And this micron refers to the wavelength of the radiation, that is wavelength of the radiation.

If you remember the units of $I_{b\lambda}(T)$ that has units of W/m^{2}μmsr, but since we have integrated this over the entire hemispherical space taking into account the variation of the angle. So, therefore, this $E_{b\lambda}(T)$ does not contain any steradian, any solid angle. So, if this is the formula and we already know through use of Planck's function; what is the expression for $I_{b\lambda}(T)$.

So, using the Planck's function for $I_{b\lambda}(T)$, I can write

$$E_{b\lambda}(T) = \frac{C_1}{\lambda^5\left\{\exp[C_2 / KT] - 1\right\}} \quad \text{W/m}^2\mu\text{m}$$

The only thing that we have done in this, to this expression is substituted the expression for the intensity from Planck's function which we have discussed in the previous class.

So, when you do that, then you get this expression for the black body radiation flux and the C_1 and C_2 are constants. So,

$$C_1 = 2\pi hc^2 = 3.743 \times 10^8$$

$$C_2 = \frac{hc}{k} = 1.4387 \times 10^8$$

So, what this expression tells us is that at any given temperature and at any given wavelength $E_{b\lambda}(T)$ can be computed. So, not only you can and so, $E_{b\lambda}(T)$ is a function both of temperature and of wavelength and therefore, if you know the wavelength and the temperature, the black body radiation flux can be computed.

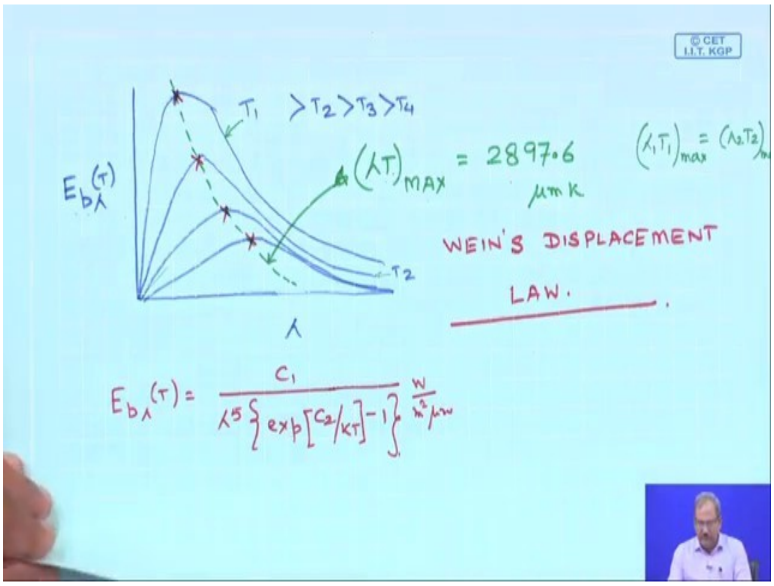
So, provided we know what is the temperature, then the spectral blackbody emissive power, I use the word spectral here purposefully to stress that the black body radiation intensity will also depend on the wavelength at which we are measuring the radiation flux.

So, all these are strongly dependent on the wavelength. Therefore, since we know now what is the relation between $E_{b\lambda}(T)$, which is the black body radiation flux with λ and T, we can plot the black body radiation function as a function of λ, the wavelength at different values of temperature.

Once again, $E_{b\lambda}(T)$ is a function of wavelength and is a function of temperature. So, keeping one constant in this case the temperature constant, I can compute the variation of $E_{b\lambda}(T)$ with λ and then, I can choose another value of temperature and again compute the same variation of $E_{b\lambda}(T)$ as a function of λ.

So, I get a series of curves, the series of curves at different values of temperature which shows the variation of the black body radiation flux as a function of wavelength given by the expression which is shown over here. So, I fix T and plot $E_{b\lambda}(T)$ as a function of λ and then, I choose a different value of temperature and plot this again. Therefore, this would result in a family of curves which would look something like this.

(Refer Slide Time: 05:56)



$(\lambda T)_{MAX} = 2897.6$ μmK

$(\lambda_1 T_1)_{max} = (\lambda_2 T_2)$

WEIN'S DISPLACEMENT LAW.

$$E_{b\lambda}(T) = \frac{C_1}{\lambda^5 \left\{ \exp\left[C_2/\lambda T\right] - 1 \right\}} \frac{W}{m^2 \mu m}$$

So, the variation of $E_{b\lambda}(T)$ as a function of wavelength will show variations like this. These are at different values of temperature T_1, T_2, T_3 etcetera and this T_1 is greater than T_2 greater than T_3 greater than T_4 and so on. So, the top one, top curve corresponds to the highest temperature, the bottom curve corresponds to the lowest temperature.

So, if these maxima the point at which the $E_{b\lambda}(T)$ is a maximum; that means, the blackboard emissive power is maximum, if these points are joined together, the locus of this line is very interesting. It is λT corresponding to the maximum value of $E_{b\lambda}(T)$

is simply a constant 2897.6 μmK. The always in radiation the wavelength is referred to as a wavelength is expressed in microns and T is in Kelvin. So, I will go through it once again.

Using the formulation that what we have written in the previous slide, let me write it once again here.

$$E_{b\lambda}(T) = \frac{C_1}{\lambda^5\left\{\exp[C_2/KT]-1\right\}} \text{ W/m}^2\mu\text{m}$$

Using this formulation, I fix temperature. Let us say I have taken the temperature to be equal to T_1 here and this is T_2, T_3 and T_4.

So, if I fix T_1, I can plot $E_{b\lambda}(T)$ as a function of λ. This is its variation. Then, I fix the temperature at T_2 slightly lower than T_1 and compute this function once again and this is what I am going to get corresponding to T_2. Then, T_3 which is less than T_2 this is the distribution of $E_{b\lambda}(T)$ as a function of λ. Interestingly, when you look at the maxima of these curves, what you see is that the product of λT corresponding to the max of $E_{b\lambda}(T)$ is a constant.

$$\left(\lambda T\right)_{MAX} = 2897.6 \ \mu\text{mK}$$

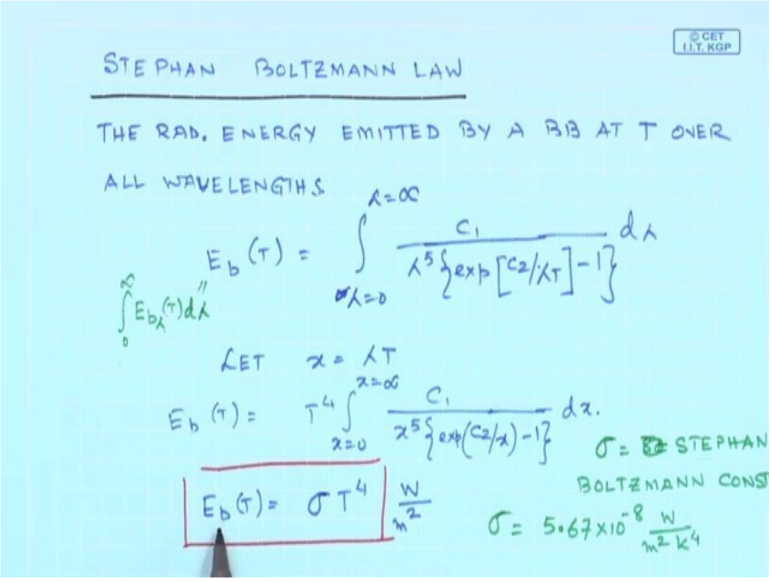
So, if you find out what is the corresponding λand what is the value of T_4; if you multiply them together and you find out what is the corresponding λ and what is the corresponding temperature T_1; so, essentially it will tell you that

$$\left(\lambda_1 T_1\right)_{MAX} = \left(\lambda_2 T_2\right)_{MAX}$$

This relation that λ corresponding to maximum $E_{b\lambda}(T)$ is a constant is known as Wein's Displacement Law.

So, what it states is that if you know the temperature, if you know the temperature, then the corresponding value of the wavelength at which $E_{b\lambda}(T)$ is a maximum can be simply obtained by this relation. We will see its use in subsequent our subsequent analysis, but let us do something which is even more which we know even better.

(Refer Slide Time: 10:28)



STEPHAN BOLTZMANN LAW

THE RAD. ENERGY EMITTED BY A BB AT T OVER ALL WAVELENGTHS.

$$E_b(T) = \int_{\lambda=0}^{\lambda=\infty} \frac{C_1}{\lambda^5\{\exp[C_2/\lambda T]-1\}} d\lambda$$

$$\overset{\prime\prime}{\int_0^\infty E_{b\lambda}(T) d\lambda}$$

LET $x = \lambda T$

$$E_b(T) = T^4 \int_{x=0}^{x=\infty} \frac{C_1}{x^5\{\exp(C_2/x)-1\}} dx.$$

$$\boxed{E_b(T) = \sigma T^4} \ \frac{W}{m^2}$$

σ = STEPHAN BOLTZMANN CONST

$\sigma = 5.67 \times 10^{-8} \frac{W}{m^2 K^4}$

So, the radiation energy that is emitted by a black body, by a black body at T over all wavelengths so, the expression that I have this is the emission radiation emission by a black body at a specific wavelength.

But we probably are more interested in to finding out what is the total energy emitted by the black body over all possible wavelengths. So, mathematically speaking, I would like to know what is the total energy emitted by the black body over a wavelength range of 0 to ∞. So, if I could find out the total if I would like to find out what is the total energy emitted by the black body.

Then, the spectral black body radiation must be integrated over all possible values of wavelength. So,

$$E_b(T) = \int_{\lambda=0}^{\lambda=\infty} \frac{C_1}{\lambda^5\left\{\exp[C_2/\lambda T]-1\right\}} d\lambda$$

So, that is how you obtain the total amount of energy emitted by a black body at an absolute temperature of T, which is what we are going to do next.

Since, what we are just saying that this must be equal to integration from 0 to ∞.

$$\int_0^\infty E_{b\lambda}(T) d\lambda"$$

that is a standard definition I have put the expression incorporating Planck's distribution.

So,

$x = \lambda T$

$$E_b(T) = T^4 \int_{x=0}^{x=\infty} \frac{C_1}{x^5\left\{\exp[C_2/x]-1\right\}} dx$$

$\sigma = 5.67 \times 10^{-8}$ W/m^{2}K^4=Stephan Boltzmann Constant

$E_b(T) = \sigma T^4$ W/m^2

So, the expression the unit for $E_{b\lambda}(T)$ is W/m^{2}µm. Since, you have integrated it over all possible wavelength. So, therefore, this one will not have this micron over here it is simply W/m^2. And this sigma, you are well aware of this is known as the Stephen Boltzmann Constant. Stephen Boltzmann Constant and the value of this σ is 5.67×10^{-8} W/m^{2}K^4 this relation obviously, then is known as the Stephen Boltzmann Law.

So, this is one of the unique laws of radiation which tells you that the total energy emitted by a black body at an absolute temperature T per unit area per unit time is a function of absolute temperature to the power 4 and the constant of proportionality is simply σ is which is a constant which can be evaluated by evaluating this definite integral and it has a numerical value of 5.67×10^{-8} W/m^{2}K^4 and this σ is known as the Stephen Boltzmann Constant.

So, what we then see is there are two interesting observations from here. The first of all that the black body radiation function, the black body that emissive power of a black body is a function of wavelength and is a function of temperature. Since it is a function of wavelength so that is why the emissive power of a blackbody, we use the adjective spectral before it to underscore the importance of wavelength while specifying the emissive power of a black body.

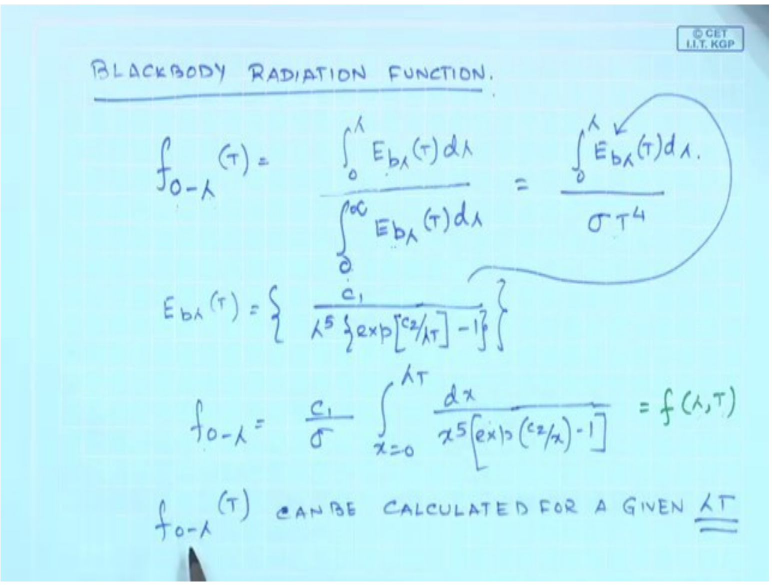
Now, when you integrate it over all possible wavelengths, what you get is the total emissive power of the black body at that given temperature. So, it is the amount of energy emitted by a black body per unit area per unit time over all wavelengths possible. So, that is why its total and I can drop the spectral from the description of the emissive power. The moment I integrate over it and the result of disintegration is the Stephen Boltzmann Law.

In some applications, you are not interested to know what is the total power emitted by the black body. You are more interested to find out what is the power emitted by the black body within a certain wavelength range let us say you would like to find out between 2 to 4 µm, how much of the energy is going to be concentrated.

So, the entire blackbody emits radiation starting at λ=0 to λ=∞, but you would not want that; you want within a specific range of λ how much of energy is going to be released, how much energy is going to be emitted by the black body as a fraction of the total energy emitted by the black body.

So, this is an important parameter which we will keep on using in our subsequent discussion. That means, the fraction of energy emitted by a black body within a specific wavelength range. So, that is that is known as the Black Body Radiation Function.

(Refer Slide Time: 18:32)



BLACKBODY RADIATION FUNCTION.

$$f_{0-\lambda}(T) = \frac{\int_0^\lambda E_{b\lambda}(T)\,d\lambda}{\int_0^\infty E_{b\lambda}(T)\,d\lambda} = \frac{\int_0^\lambda E_{b\lambda}(T)\,d\lambda.}{\sigma T^4}$$

$$E_{b\lambda}(T) = \left\{ \frac{c_1}{\lambda^5 \left\{ \exp\left[c_2/\lambda T \right] - 1 \right\}} \right\}$$

$$f_{0-\lambda} = \frac{c_1}{\sigma} \int_{x=0}^{\lambda T} \frac{dx}{x^5 \left[\exp\left(c_2/x \right) - 1 \right]} = f(\lambda, T)$$

$f_{0-\lambda}(T)$ CAN BE CALCULATED FOR A GIVEN $\underline{\underline{\lambda T}}$

So, we would see what is a Blackbody Radiation Function? So, Blackbody Radiation Function as I have expressed to you; so, it is between a wavelength from 0 to λ at a given temperature is the amount of energy emitted by the black body between 0 to λ divided by 0 to ∞; that means, the total energy emitted by the black body at that a given temperature.

So, once again, the fraction the black body radiation function, the fraction of energy which is emitted by the black body within a wavelength range from 0 to λ is nothing but the amount of energy emitted by the black body between 0 to δ divided by the amount of energy emitted by the black body over 0 to ∞. So, this is the total energy and this is the amount of energy within a fixed wavelength range.

$$f_{0-\lambda}(T)=\frac{\int_0^\lambda E_{b\lambda}(T)d\lambda}{\int_0^\infty E_{b\lambda}(T)d\lambda}=\frac{\int_0^\lambda E_{b\lambda}(T)d\lambda}{\sigma T^4}$$

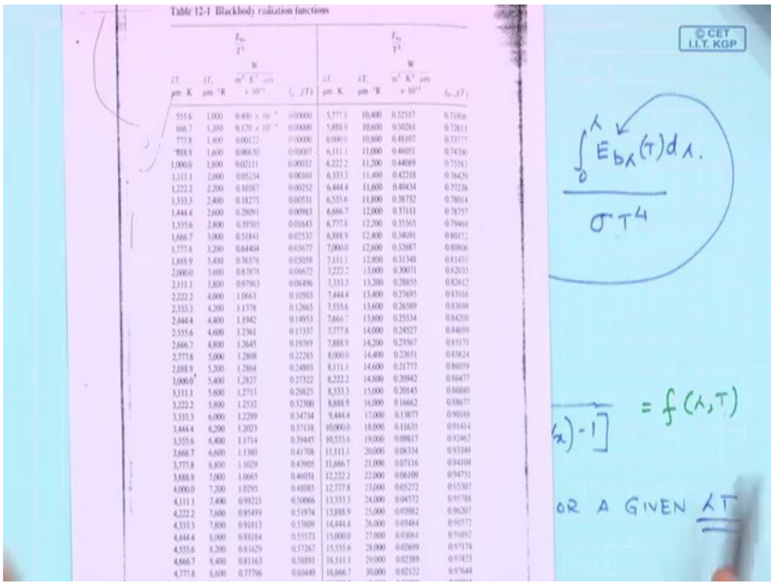
$$E_{b\lambda}(T)=\frac{C_1}{\lambda^5\left\{\exp[C_2/\lambda T]-1\right\}}$$

$$f_{0-\lambda}=\frac{C_1}{\sigma}\int_{x=0}^{\lambda T}\frac{dx}{x^5\left\{\exp[C_2/x]-1\right\}}=f\left(\lambda,T\right)$$

So, this integral can be evaluated and $f_{0-\lambda}(T)$ can be calculated for a given λT. So, you would I guess in the idea behind this is very clear to you. I am trying to find out the fraction of energy which is emitted by the black body that is going to be concentrated within a wavelength range of 0 to λ.

So, if this is a fraction, then this is going to be the energy which is going to be within 0 to λby the total energy emitted by the black body. Total energy emitted by the black body is nothing but is σT^4 and this can be substituted by the Planck's function as we have done before. When you incorporate that you get the fraction in this form. This is again an integral equation which can be evaluated provided you specify λ and T. So, once you specify λ and T, this fraction can be evaluated.

(Refer Slide Time: 22:50)



$$\frac{\int_0^{\lambda} E_{b\lambda}(T)\,d\lambda}{\sigma T^4}$$

$x)-1]$ $= f(\lambda, T)$

OR A GIVEN λT

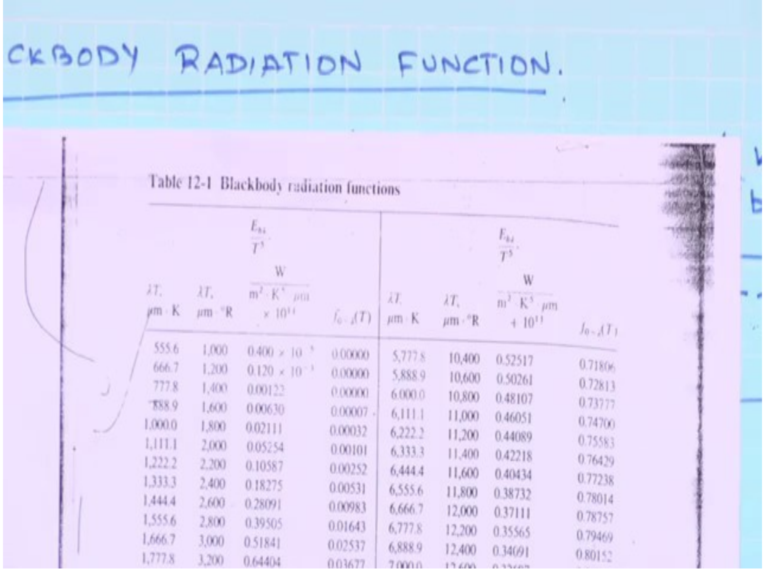
Table 12-1 Blackbody radiation functions

λT, µm · K	λT, µm · °R	$\frac{E_{b\lambda}}{T^5}$, $\frac{W}{m^2 \cdot K^5 \cdot \mu m}$ × 10^{11}	$f_{0-\lambda}(T)$	λT, µm · K	λT, µm · °R	$\frac{E_{b\lambda}}{T^5}$, $\frac{W}{m^2 \cdot K^5 \cdot \mu m}$ × 10^{11}	$f_{0-\lambda}(T)$
555.6	1,000	0.400 × 10^{-5}	0.00000	5,777.8	10,400	0.52517	0.71806
666.7	1,200	0.120 × 10^{-3}	0.00000	5,888.9	10,600	0.50261	0.72813
777.8	1,400	0.00122	0.00000	6,000.0	10,800	0.48107	0.73777
888.9	1,600	0.00630	0.00007	6,111.1	11,000	0.46051	0.74700
1,000.0	1,800	0.02111	0.00032	6,222.2	11,200	0.44089	0.75583
1,111.1	2,000	0.05254	0.00101	6,333.3	11,400	0.42218	0.76429
1,222.2	2,200	0.10587	0.00252	6,444.4	11,600	0.40434	0.77238
1,333.3	2,400	0.18275	0.00531	6,555.6	11,800	0.38732	0.78014
1,444.4	2,600	0.28091	0.00983	6,666.7	12,000	0.37111	0.78757
1,555.6	2,800	0.39505	0.01643	6,777.8	12,200	0.35565	0.79469
1,666.7	3,000	0.51841	0.02537	6,888.9	12,400	0.34091	0.80152
1,777.8	3,200	0.64404	0.03677	7,000.0	12,600	0.32687	0.80806
1,888.9	3,400	0.76578	0.05059	7,111.1	12,800	0.31348	0.81433
2,000.0	3,600	0.87878	0.06672	7,222.2	13,000	0.30071	0.82035
2,111.1	3,800	0.97963	0.08496	7,333.3	13,200	0.28855	0.82612
2,222.2	4,000	1.0663	0.10503	7,444.4	13,400	0.27695	0.83166
2,333.3	4,200	1.1378	0.12665	7,555.6	13,600	0.26589	0.83698
2,444.4	4,400	1.1942	0.14953	7,666.7	13,800	0.25534	0.84209
2,555.6	4,600	1.2361	0.17337	7,777.8	14,000	0.24527	0.84699
2,666.7	4,800	1.2645	0.19789	7,888.9	14,200	0.23567	0.85171
2,777.8	5,000	1.2808	0.22285	8,000.0	14,400	0.22651	0.85624
2,888.9	5,200	1.2864	0.24803	8,111.1	14,600	0.21777	0.86059
3,000.0	5,400	1.2827	0.27322	8,222.2	14,800	0.20942	0.86477
3,111.1	5,600	1.2713	0.29825	8,333.3	15,000	0.20145	0.86880
3,222.2	5,800	1.2532	0.32300	8,888.9	16,000	0.16662	0.88677
3,333.3	6,000	1.2299	0.34734	9,444.4	17,000	0.13877	0.90168
3,444.4	6,200	1.2023	0.37118	10,000.0	18,000	0.11635	0.91414
3,555.6	6,400	1.1714	0.39445	10,555.6	19,000	0.09817	0.92462
3,666.7	6,600	1.1380	0.41708	11,111.1	20,000	0.08334	0.93349
3,777.8	6,800	1.1029	0.43905	11,666.7	21,000	0.07116	0.94104
3,888.9	7,000	1.0665	0.46031	12,222.2	22,000	0.06109	0.94751
4,000.0	7,200	1.0295	0.48085	12,777.8	23,000	0.05272	0.95307
4,111.1	7,400	0.99221	0.50066	13,333.3	24,000	0.04572	0.95788
4,222.2	7,600	0.95499	0.51974	13,888.9	25,000	0.03982	0.96207
4,333.3	7,800	0.91813	0.53809	14,444.4	26,000	0.03484	0.96572
4,444.4	8,000	0.88184	0.55573	15,000.0	27,000	0.03061	0.96892
4,555.6	8,200	0.84629	0.57267	15,555.6	28,000	0.02699	0.97174
4,666.7	8,400	0.81163	0.58891	16,111.1	29,000	0.02389	0.97423
4,777.8	8,600	0.77796	0.60449	16,666.7	30,000	0.02122	0.97644

So, this fraction is calculated for a given λ times T and you would see that the results of the fraction is provided in a table.

So, in that table the total emissive power of a black body within which is contained within the wavelength range is provided as a function of T and as a function of λ. So, one you must keep in mind that this T is always going to be in Kelvin and λ is going to be in micron.

(Refer Slide Time: 23:28)



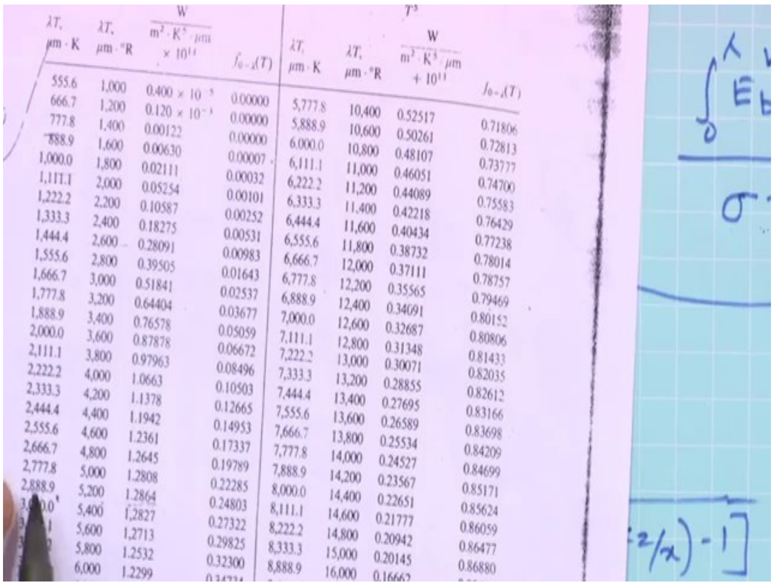
CKBODY RADIATION FUNCTION.

Table 12-1 Blackbody radiation functions

λT, µm · K	λT, µm · °R	$\frac{E_{b\lambda}}{T^5}$, $\frac{W}{m^2 \cdot K^5 \cdot \mu m}$ × 10^{11}	$f_{0-\lambda}(T)$	λT, µm · K	λT, µm · °R	$\frac{E_{b\lambda}}{T^5}$, $\frac{W}{m^2 \cdot K^5 \cdot \mu m}$ × 10^{11}	$f_{0-\lambda}(T)$
555.6	1,000	0.400 × 10^{-5}	0.00000	5,777.8	10,400	0.52517	0.71806
666.7	1,200	0.120 × 10^{-3}	0.00000	5,888.9	10,600	0.50261	0.72813
777.8	1,400	0.00122	0.00000	6,000.0	10,800	0.48107	0.73777
888.9	1,600	0.00630	0.00007	6,111.1	11,000	0.46051	0.74700
1,000.0	1,800	0.02111	0.00032	6,222.2	11,200	0.44089	0.75583
1,111.1	2,000	0.05254	0.00101	6,333.3	11,400	0.42218	0.76429
1,222.2	2,200	0.10587	0.00252	6,444.4	11,600	0.40434	0.77238
1,333.3	2,400	0.18275	0.00531	6,555.6	11,800	0.38732	0.78014
1,444.4	2,600	0.28091	0.00983	6,666.7	12,000	0.37111	0.78757
1,555.6	2,800	0.39505	0.01643	6,777.8	12,200	0.35565	0.79469
1,666.7	3,000	0.51841	0.02537	6,888.9	12,400	0.34091	0.80152
1,777.8	3,200	0.64404	0.03677				

So, if you look at the table now, the table here shows you; the table here shows you the value of λT in micron and this is $\frac{E_{b\lambda}}{T^5}$. This is how the fraction is shown. So, you know what is λT. Let us say you pick a value of λT to be over here. λT is 2777.8. So, any combination of λ and T which gives the products to be about 2777, it would give you a value of the f as 0.22.

(Refer Slide Time: 24:15)

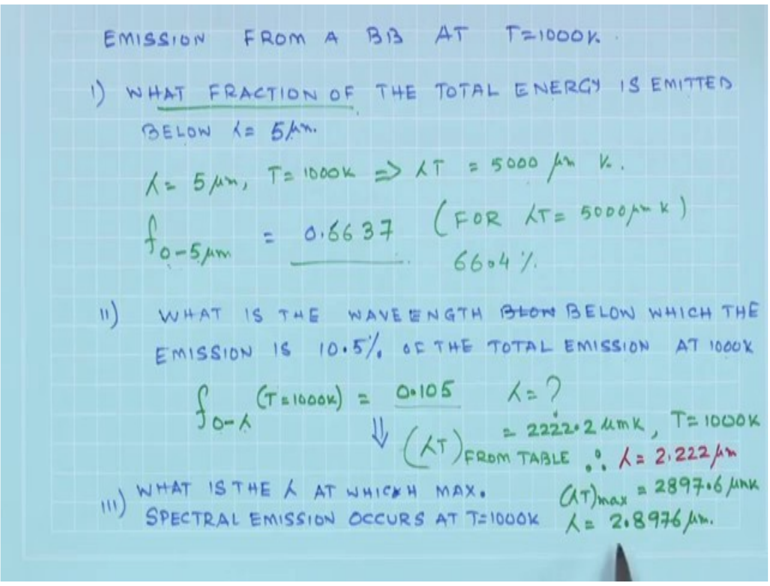


λT, μm · K	λT, μm · °R	$\frac{W}{m^2 \cdot K^5 \cdot \mu m} \times 10^{11}$	$f_{0-\lambda}(T)$	λT, μm · K	λT, μm · °R	T^5 $\frac{W}{m^2 \cdot K^5 \cdot \mu m} + 10^{11}$	$f_{0-\lambda}(T)$
555.6	1,000	0.400×10^{-5}	0.00000	5,777.8	10,400	0.52517	0.71806
666.7	1,200	0.120×10^{-3}	0.00000	5,888.9	10,600	0.50261	0.72813
777.8	1,400	0.00122	0.00000	6,000.0	10,800	0.48107	0.73777
888.9	1,600	0.00630	0.00007	6,111.1	11,000	0.46051	0.74700
1,000.0	1,800	0.02111	0.00032	6,222.2	11,200	0.44089	0.75583
1,111.1	2,000	0.05254	0.00101	6,333.3	11,400	0.42218	0.76429
1,222.2	2,200	0.10587	0.00252	6,444.4	11,600	0.40434	0.77238
1,333.3	2,400	0.18275	0.00531	6,555.6	11,800	0.38732	0.78014
1,444.4	2,600	0.28091	0.00983	6,666.7	12,000	0.37111	0.78757
1,555.6	2,800	0.39505	0.01643	6,777.8	12,200	0.35565	0.79469
1,666.7	3,000	0.51841	0.02537	6,888.9	12,400	0.34091	0.80152
1,777.8	3,200	0.64404	0.03677	7,000.0	12,600	0.32687	0.80806
1,888.9	3,400	0.76578	0.05059	7,111.1	12,800	0.31348	0.81433
2,000.0	3,600	0.87878	0.06672	7,222.2	13,000	0.30071	0.82035
2,111.1	3,800	0.97963	0.08496	7,333.3	13,200	0.28855	0.82612
2,222.2	4,000	1.0663	0.10503	7,444.4	13,400	0.27695	0.83166
2,333.3	4,200	1.1378	0.12665	7,555.6	13,600	0.26589	0.83698
2,444.4	4,400	1.1942	0.14953	7,666.7	13,800	0.25534	0.84209
2,555.6	4,600	1.2361	0.17337	7,777.8	14,000	0.24527	0.84699
2,666.7	4,800	1.2645	0.19789	7,888.9	14,200	0.23567	0.85171
2,777.8	5,000	1.2808	0.22285	8,000.0	14,400	0.22651	0.85624
2,888.9	5,200	1.2864	0.24803	8,111.1	14,600	0.21777	0.86059
[illegible]	5,400	1.2827	0.27322	8,222.2	14,800	0.20942	0.86477
[illegible]	5,600	1.2713	0.29825	8,333.3	15,000	0.20145	0.86880
[illegible]	5,800	1.2532	0.32300	8,888.9	16,000	0.16662	[illegible]
[illegible]	6,000	1.2299	[illegible]				

So, let us write it over here if I write it in this so, for a value of λT equal to 2777, the value of the fraction is 0.2285. So, for λT equals 2777.8, the value of f is going to be equal 0.22. So, which means that you choose some value of λ and T, the product comes close to 2777.8. So, if that is the case, then within that wavelength range at that given temperature, you can find out what is the fraction of energy that is emitted by the black body is contained within the wavelength.

And I think it would be more clear to you once we solve a problem using the table and this f the black body radiation function, the table of blackbody radiation function you can refer to and find out that given the value of λ and T what is the fraction that is going to be, what is a black body radiation function. So, we will quickly solve 1 problem and I think that would clarify any remaining doubts that you may have. But fundamentally, we are finding out the fraction of energy which is contained within 0 to some λ at a given temperature for the blackbody emissions. So, let us quickly solve 1 problem and then we will move on.

(Refer Slide Time: 25:52)



EMISSION FROM A BB AT T=1000K.

I) WHAT FRACTION OF THE TOTAL ENERGY IS EMITTED BELOW $\lambda = 5\mu m$.

$\lambda = 5\mu m$, $T = 1000K \Rightarrow \lambda T = 5000\ \mu m\ K$.

$f_{0-5\mu m} = \underline{0.6637}$ (FOR $\lambda T = 5000\mu m\ K$)

66.4 %

II) WHAT IS THE WAVELENGTH ~~BLOW~~ BELOW WHICH THE EMISSION IS 10.5% OF THE TOTAL EMISSION AT 1000K

$f_{0-\lambda}(T=1000K) = \underline{0.105}$ $\lambda = ?$

$\Downarrow$ $(\lambda T)_{\text{FROM TABLE}} = 2222.2\ \mu m K$, $T = 1000K$

$\therefore \lambda = 2.222\mu m$

III) WHAT IS THE λ AT WHICH MAX. SPECTRAL EMISSION OCCURS AT T=1000K

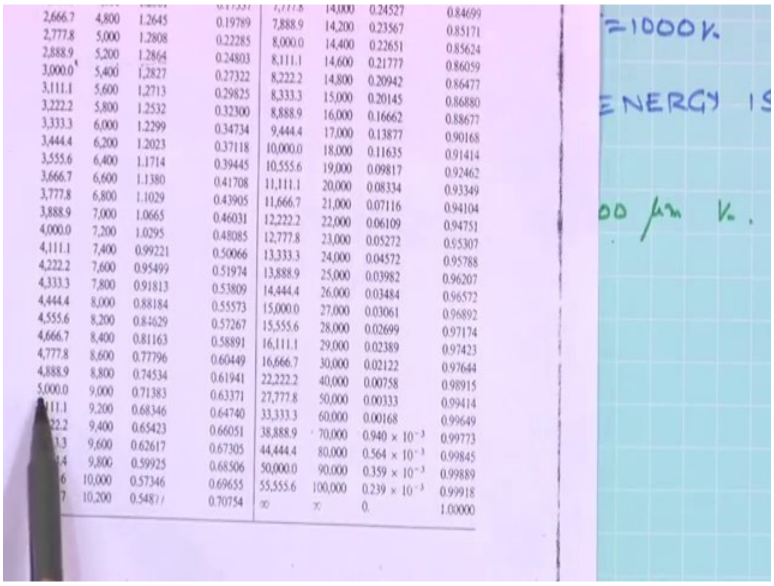
$(\lambda T)_{max} = 2897.6\ \mu m K$

$\lambda = 2.8976\ \mu m$.

So, what the problem says is that the emission is from a surface, from a black body at T=1000 K. The first part is what fraction of the total energy is emitted below λ=5µm. So, what we need to do then is this my λ=5µm, T is 1000 K. So, λT is 5000 µm.K.

So, when we would say fraction. So, I would like to find out 0 to 5 µm. The value of the black body radiation function between 0 to 5 µm, fundamentally what this tells us is f 0 to 5 is the fraction of the total energy that is emitted by a black body at a given temperature between the range 0 to 5 µm. So, from the adjoining table, I need to find out what is the value of f of 0 to 5 µm.

(Refer Slide Time: 27:43)



[illegible]	[illegible]	[illegible]	[illegible]	7,777.8	14,000	0.24527	0.84699
2,666.7	4,800	1.2645	0.19789	7,888.9	14,200	0.23567	0.85171
2,777.8	5,000	1.2808	0.22285	8,000.0	14,400	0.22651	0.85624
2,888.9	5,200	1.2864	0.24803	8,111.1	14,600	0.21777	0.86059
3,000.0	5,400	1.2827	0.27322	8,222.2	14,800	0.20942	0.86477
3,111.1	5,600	1.2713	0.29825	8,333.3	15,000	0.20145	0.86880
3,222.2	5,800	1.2532	0.32300	8,888.9	16,000	0.16662	0.88677
3,333.3	6,000	1.2299	0.34734	9,444.4	17,000	0.13877	0.90168
3,444.4	6,200	1.2023	0.37118	10,000.0	18,000	0.11635	0.91414
3,555.6	6,400	1.1714	0.39445	10,555.6	19,000	0.09817	0.92462
3,666.7	6,600	1.1380	0.41708	11,111.1	20,000	0.08334	0.93349
3,777.8	6,800	1.1029	0.43905	11,666.7	21,000	0.07116	0.94104
3,888.9	7,000	1.0665	0.46031	12,222.2	22,000	0.06109	0.94751
4,000.0	7,200	1.0295	0.48085	12,777.8	23,000	0.05272	0.95307
4,111.1	7,400	0.99221	0.50066	13,333.3	24,000	0.04572	0.95788
4,222.2	7,600	0.95499	0.51974	13,888.9	25,000	0.03982	0.96207
4,333.3	7,800	0.91813	0.53809	14,444.4	26,000	0.03484	0.96572
4,444.4	8,000	0.88184	0.55573	15,000.0	27,000	0.03061	0.96892
4,555.6	8,200	0.84629	0.57267	15,555.6	28,000	0.02699	0.97174
4,666.7	8,400	0.81163	0.58891	16,111.1	29,000	0.02389	0.97423
4,777.8	8,600	0.77796	0.60449	16,666.7	30,000	0.02122	0.97644
4,888.9	8,800	0.74534	0.61941	22,222.2	40,000	0.00758	0.98915
5,000.0	9,000	0.71383	0.63371	27,777.8	50,000	0.00333	0.99414
111.1	9,200	0.68346	0.64740	33,333.3	60,000	0.00168	0.99649
22.2	9,400	0.65423	0.66051	38,888.9	70,000	0.940×10^{-3}	0.99773
3.3	9,600	0.62617	0.67305	44,444.4	80,000	0.564×10^{-3}	0.99845
4	9,800	0.59925	0.68506	50,000.0	90,000	0.359×10^{-3}	0.99889
6	10,000	0.57346	0.69655	55,555.6	100,000	0.239×10^{-3}	0.99918
7	10,200	0.54877	0.70754	∞	∞	0.	1.00000

=1000 K.

ENERGY IS

00 μm K.

So, for which case λT is 5000 μm.K. So, if you look at the value over here for a value of λT to be equal to 5000 μm.K , the value of f is 0.66371. So, from the table, I read that this is equal to 0.6637 for λT equals 5000 μm.K which simply tells me that 66.4 % or 0.664; the fraction 66.4 % of the total energy is emitted by the black body at 1000 K, below λ=5 μm; that means, this is the value of the of the fraction.

Second question is what is the wavelength below which the emission is 10.5 % of the total emission at 1000 K? So, it is reverse of the previous problem. In this case f $_{0\text{-}\lambda}$ at a given temperature of 1000 K is provided to be 0.105. We need to find out what is the value of λ. So, once again the emission is 10.5 % of the total emission at 1000 K. So the fraction which of energy which is contained between 0 to the unknown λ at 1000 K is specified to be 0.105. We need to find out what is the value of λ for such a case.

Once again, we look at the table and when we look at the table, now we have to look at λ to be the fraction to be 0.105. So, when you look at the fraction to be 0.105, you read the value of λT to be 2222 μm.K. So, this refers to the value of λT. And since, you know the temperature to be equals 1000 K. So, therefore, your λis simply going to be 2.222 μm.

So, at 1000 K, the emission is going to be
10.5 % of the total emission at and wavelength of 2.2 μm and the third one is. So, what is the wavelength? What is the λ at which the maximum spectral emission occurs at T=1000K .

So, the third question is what is the wavelength at which you are going to get the maximum spectral emission? The moment I say spectral that means, its wavelength dependent so, the value of the temperature is provided to you; you have to find out what is the wavelength at which the emission is going to be maximum.

So, of course the law that is that we need to use for this is Wein's Displacement Law. The Wein's Displacement Law simply states that λT is a constant when we are talking about the maximum spectral radiation intensity at a given temperature. Since the temperature over here is provided we need to find out what is the λ in this case. So, this problem can simply be solved using Wein's Law as λT corresponding to the maximum emission is 2897.6 μm.K. So, for T= 1000K, this λ would simply be equals to 2.8976 μm. So, therefore, if your temperature is at 1000 K, then the maximum the spectral maximum of emission from a black body at 1000 K will take place at a wavelength of 2.8 μm.

So, what we have discussed, if I would like to do a summary of this is that looking at the black body emission when incorporating the Planck's formula in it, we have derived the Wein's Displacement Law which shows that the product of λT corresponding to maximum spectral emission from a black body is a constant which is 2897.6 μm.K.
Secondly, the spectral emission if you would like to convert it to the total emission from a black body over all possible wavelengths that would give rise to the well known Stephen Boltzmann Law where the proportionality between the emission and the temperature is E is proportional to T^4 and the proportionality constant is Planck's constant.

Next, we wanted to know the fraction of energy which is emitted between a certain wavelength range as compared to the total energy emitted by the black body overall wavelength range. So, this black body radiation function is evaluated for different values of the product of λ and T different values of the product of the wavelength and temperature.

So, these black body radiation functions provide us with the knowledge of how much of energy is going to be emitted by a black body at a given temperature within a specific range of wavelengths. And we have solved the problem numerical problem which I think would clarify any doubts that you may have in the use of Wein's Displacement Law, the Stephen Boltzmann Law and the Blackbody Radiation Function.